
(M^2)



KAIST

가

1988

, KAIST

MATHLETTER

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2003 8 ,

1		7
1.1	8
1.2	21
2		37
2.1	38
2.2	47
2.3	59
3		67
3.1	68
3.2	80
3.3	97
3.4 가	110
4		123
4.1	124

4.2	138
5		153
5.1	154
5.2	167
5.3	가	176

1

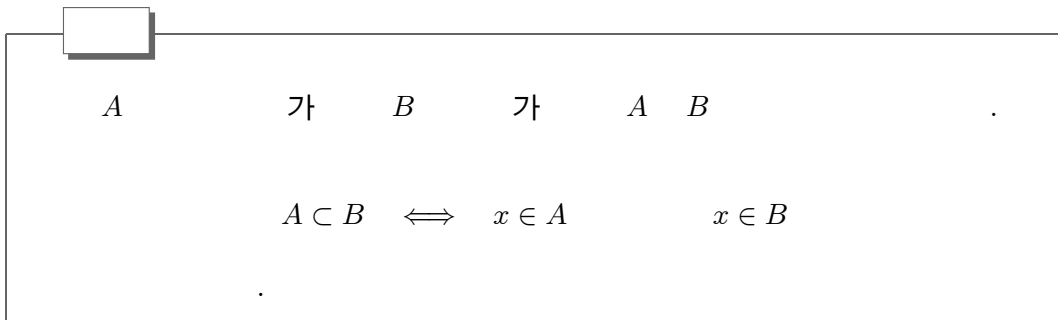
1.1



\in \subset =가 . $x \in A, A \subset B,$
 $C = D$ 가 $A \cup B, C \cap D$
 가 ,

가 ' _ ' , ' _ ' ,
 ' _ ' , ' _ ' ,

가



' \iff '

Q가 , 'P가
 Q가 ' $P \Rightarrow Q$.

- (1) $A \subset A$
- (2) $A \subset B \quad B \subset C \quad A \subset C$
- (3) $\emptyset \subset A$

■ (1) $A \subset A$ 가 ,
 $A \subset A$. ,

$$x \in A \quad x \in A \iff A \subset A$$

(2) $A \subset B \quad B \subset C$, $A \subset B$
 , $B \subset C$. , $A \subset C$
 가 , $A \subset C$.

$$A \subset B \iff x \in A \quad x \in B$$

$$B \subset C \iff x \in B \quad x \in C$$

,
 $x \in A \quad x \in B$ 가 , $x \in B \quad x \in C$ 가
 $x \in A \quad x \in C$ 가 . ,

$$x \in A \quad x \in C \iff A \subset C$$

(3) $\emptyset \subset A$, $\emptyset \subset A$ 가
 . ,

$$\emptyset \subset A \iff x \in \emptyset \quad x \in A$$

\emptyset 가 $x \in \emptyset$ x 가 , $x \in A$ 가
 .
 가 , \emptyset A
 .
 . □

□

A 가 B 가 , B 가 A 가
 , $A \subset B$ 가 .

$$A = B \iff A \subset B \quad B \subset A$$

- (1) $A = A$
- (2) $A = B$ $B = A$ 가 .
- (3) $A = B$ $B = C$ $A = C$ 가 .

■ (1) $A \subset A$ $A \supset A$, $A = A$

(2) $A = B$

$$A \subset B \quad B \subset A$$

$$B \subset A \quad A \subset B$$

가 . $B = A$.
 (3) $A = B$ $B = C$,

$$A \subset B, B \subset A, B \subset C, C \subset B$$

$$A \subset B \quad B \subset C \quad , \quad (2)$$

$$A \subset C \quad . C \subset B \quad B \subset A \quad C \subset A$$

$$A \subset C \quad C \subset A$$

, $A = C$. □

■

?

(1) : $A \cup B = \{x \mid x \in A \quad x \in B\}$

(2) : $A \cap B = \{x \mid x \in A \quad x \in B\}$

(3) : $A - B = \{x \mid x \in A \quad x \notin B\}$

(4) : $A^c = \{x \in U \mid x \notin A\}$

: $x \in A \cup B \iff x \in A \quad x \in B$

: $x \in A \cap B \iff x \in A \quad x \in B$

: $x \in A - B \iff x \in A \quad x \notin B$

: $x \in A^c \iff x \notin A$

$$(1) A - B = A \cap B^c$$

$$(2) A^c = U - A$$

$$(3) (A^c)^c = A$$

■ (1)

$$\begin{aligned} x \in A - B &\iff x \in A \quad x \notin B \\ &\iff x \in A \quad x \in B^c \\ &\iff x \in A \cap B^c \end{aligned}$$

(2) $x \in U$ 가
가

$$\begin{aligned} x \in A^c &\iff x \in U \quad x \notin A \\ &\iff x \in U \cap A^c = U - A \end{aligned}$$

(1)

(3)

$$x \in (A^c)^c \iff \neg(x \in A^c) \iff \neg\neg(x \in A)$$

\neg ' ' ,

$$x \in A$$

□

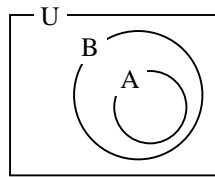
(De Morgan)

$$(A \cup B)^c = A^c \cap B^c, (A \cap B)^c = A^c \cup B^c$$

'A B ' 'A가 B가 '가
, ' ' '가

$$A \subset B \quad : \quad B^c \subset A^c, A \cap B = A, A \cup B = B, A - B = \emptyset$$

, $A \subset B$



, $A - B = \emptyset$

 $A \subset B \implies A - B = \emptyset$: A B 가 , A
 B 가 $A \cap B^c = \emptyset$.

$$A - B = \emptyset$$

$A - B = \emptyset \implies A \subset B$: □

1.1.1

A A $P(A)$
 2^A $\cdot A \subset B$ $2^A \subset 2^B$ \cdot

□

2^A , A 가

$$|2^A| = 2^{|A|}$$

가

■

$X \in 2^A$

$X \in 2^B$ 가

$$X \text{가 } A \implies X \text{가 } B$$

$\cdot A \subset B$

가

,

$$X \subset A \quad A \subset B \implies X \subset B$$

□

□

A

B

A 가 B

가?

,

1.1.2

$U = 10$. U A, B 가
 $A \cap B = \{2, 4, 6\}$, $A \cup B = \{2, 3, 4, 5, 6, 8\}$. $B = \{2, 3, 4, 6\}$
 A .



가

A

가

 $A \cap B, A \cup B, B$


$$A = (A \cap B) \cup (A - B) = (A \cap B) \cup [(A \cup B) - B]$$

A

$$A = \{2, 4, 5, 6, 8\}$$

◇



$$|A \cup B| = 7, |A \cup C| = 10, |B \cup C| = 12 \quad |A \cup B \cup C| = 13$$

,

$$|(A \cap B) \cup (A \cap C) \cup (B \cap C)|$$

1.1.3

(1) $[A^c \cup (A \cap B^c)]^c$

(2) $[A \cap (A^c \cup B)] \cup [B \cap (B \cup C)]$

■ (1)

$$\begin{aligned}
 [A^c \cup (A \cap B^c)]^c &= A \cap (A \cap B^c)^c && (\quad) \\
 &= A \cap (A^c \cup B) && (\quad) \\
 &= (A \cap A^c) \cup (A \cap B) && (\quad) \\
 &= \emptyset \cup (A \cap B) = A \cap B
 \end{aligned}$$

(2)

$$\begin{aligned}
 [A \cap (A^c \cup B)] \cup [B \cap (B \cup C)] \\
 &= [(A \cap A^c) \cup (A \cap B)] \cup [(B \cap B) \cup (B \cap C)] && (\quad) \\
 &= [\emptyset \cup (A \cap B)] \cup [B \cup (B \cap C)] \\
 &= (A \cap B) \cup B = B && (\quad)
 \end{aligned}$$

◇

○

□

$$(A \cup B) \cap (A^c \cup B)$$

1.1.1

$$A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A$$

1.1.2

(1)

(2)

1.1.3

$$(A^c \cap B)^c \cap (A \cup B)$$

1.1.4

$$(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$$

1.1.5

$$A = \{x \mid x = 2m, m \in \mathbb{Z}\}, \quad B = \{y \mid y = 2n + 4, n \in \mathbb{Z}\}$$

$$A = B$$

1.1.6

$$(A \cup B) \cap (A^c \cup C) = (A \cap C) \cup (A^c \cap B)$$



1.1.7

$A = \{m + 2n \mid 3m + 2n \leq 10, m, n \in \mathbb{Z}\}$,
 A 의 원소를 구하시오.



1.1.8

 A

$$a \in A \implies \frac{a}{a-1} \in A \quad (a \neq 1)$$

$$\frac{1}{2} \in A \text{ 이므로, } \frac{1}{2} \text{ 가 } A \text{ 의 원소이다.}$$



1.1.9

$$A = \{a_1, a_2, a_3, a_4\}, \quad B = \{\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}\}$$

, $a_1 + a_2 = 13$, $A \cap B = \{a_1, a_2\}$, $a_1 < a_2 < a_3 < a_4$ 이고,
 $a_1 + a_2 + a_3 + a_4 = 28$ 이라 하자.



1.1.10

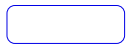
A, B, C, D, E 가 집합 U 의 부분집합일 때,

$$[(B \cap C) \cup (A \cap E)] \cap [(B \cap C) \cup A^c]$$

$$\cup [[(C \cap D^c) \cup (A \cap E^c)] \cap [(C \cap D^c) \cup A^c]]$$

$$\cup [(B^c \cup C)^c \cup (C \cup D)^c]$$

이 식을 $Y \cup Z^c$ 형태로 나타내시오. 여기서 Y, Z 는 A, B, C, D, E 의 집합 연산으로 나타내시오.



1.1.1

“ N 가 , 10

.”

N 20

? 가

N

1.2

0 1 가 0 1 ,
 . 0 1 ,



0 1 (no) (yes), (false) (true)
 ,

(+) (·)
 0 1 0 1 .
 가 .

(complement, NOT) 가 (-, bar) .
 :

$$\bar{0} = 1, \quad \bar{1} = 0$$

(OR) + . :

$$1 + 1 = 1, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 0 + 0 = 0$$

0 0 0 1 .

(AND) . .

:

$$1 \cdot 1 = 1, \quad 1 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 0 \cdot 0 = 0$$



$$0 \quad 1 \quad , \quad 0 \quad 1$$

$$F(x) = \overline{(\bar{x} + x)}, \quad G(x, y) = x \cdot y + x, \quad H(x, y, z) = x \cdot z + \bar{y}$$

3

x, y, z

$$F(x, y, z) = x \cdot y + \bar{z}$$

x	y	z	$x \cdot y$	\bar{z}	$F(x, y, z)$
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	1	1



4

x, y, z

$$F(x, y, z) = x + y \cdot z + \overline{(y + z)}$$



가 가

1. (double complement)

$$\overline{\overline{x}} = x$$

2. (idempotent)

$$x + x = x, \quad x \cdot x = x$$

3. (identity)

$$x + 0 = x, \quad x \cdot 1 = x$$

4. (dominance)

$$x + 1 = 1, \quad x \cdot 0 = 0$$

5. (commutative law)

$$x + y = y + x, \quad x \cdot y = y \cdot x$$

6. (distributive law)

$$x + y \cdot z = (x + y) \cdot (x + z), \quad x \cdot (y + z) = x \cdot y + x \cdot z$$

7. (De Morgan's law)

$$\overline{x \cdot y} = \overline{x} + \overline{y}, \quad \overline{x + y} = \overline{x} \cdot \overline{y}$$

$$x \cdot (x + y) = x$$



.

$$\begin{aligned}
 x \cdot (x + y) &= x \cdot x + x \cdot y \\
 &= x + x \cdot y \\
 &= x \cdot 1 + x \cdot y \\
 &= x \cdot (1 + y) \\
 &= x \cdot 1 = x
 \end{aligned}$$

◇

6

.

$$x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$$



- 가 가 . ,
- 가 가 가 .
- 가 가 .
1. 가 ?
2. ?

2
 ,
 . $x + y$ $y + x$ 가
 1 ?
 가
 ()

$$x \cdot y + \bar{x} \cdot \bar{y}$$

$$(x + y) \cdot (\bar{x} + \bar{y})$$

7

$F(x, y, z)$

x	y	z	$F(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

F 1 . x, y, z 가 1, 1, 0 0, 1, 0
 F 1 .

$$F(x, y, z) = x \cdot y \cdot \bar{z} + \bar{x} \cdot y \cdot \bar{z}$$

◇

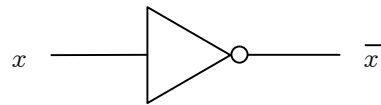
1
, 0
.

8

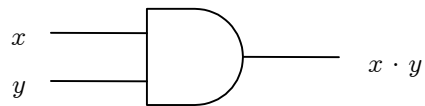
■ (gate)

가 가

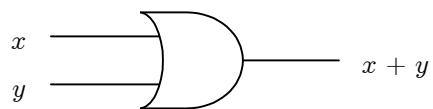
- (inverter, NOT)



- AND (AND gate)

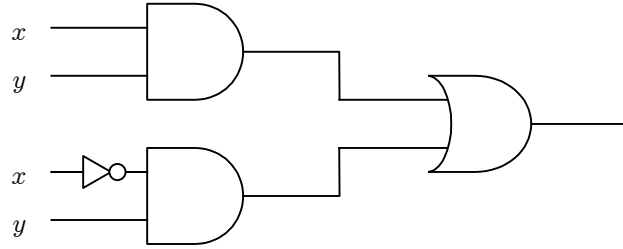


- OR (OR gate)



9

$$x \cdot y + \bar{x} \cdot y$$



. NAND

gate NOR gate

가

AND gate OR gate가

NAND

gate NOR gate

?

, AND gate, OR gate

NAND gate, NOR gate

?



가

가

가

?

	\bar{y}	y
\bar{x}	$F(0,0)$	$F(0,1)$
x	$F(1,0)$	$F(1,1)$

$x \cdot y + \bar{x} \cdot y$

가 $F(x, y) =$

	\bar{y}	y
\bar{x}	0	1
x	0	1

1

가

1

	\bar{y}	y
\bar{x}	0	1
x	0	1

$F(x, y) = x \cdot y + \bar{x} \cdot y = y$

	\bar{y}	y
\bar{x}	1	1
x	1	0

	\bar{y}	y
\bar{x}	1	1
x	1	0

$\bar{x} \cdot \bar{y} + \bar{x} \cdot y = \bar{x}$,

$\bar{x} \cdot \bar{y} + x \cdot \bar{y} = \bar{y}$

$\bar{x} \cdot \bar{y} + \bar{x} \cdot y + x \cdot \bar{y} = (\bar{x} \cdot \bar{y} + \bar{x} \cdot y) + (\bar{x} \cdot \bar{y} + x \cdot \bar{y}) = \bar{x} + \bar{y}$

가 2

, 가 3, 4

() ()

2^n

가 3

	00	01	11	10
0				
1				

$(0, 1), (1, 1), (1, 0)$, $(y, z) = (0, 0),$
 $. 01 11 10$,
 가

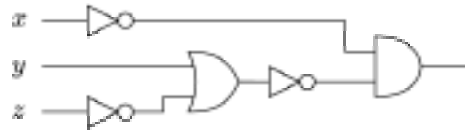
	00	01	11	10		00	01	11	10	
0	1	1	0	1	0	1	1	0	1	$\bar{y} + \bar{x} \cdot \bar{z}$
1	1	1	0	0	1	1	1	0	0	

	00	01	11	10		00	01	11	10	
0	1	1	0	1	0	1	1	0	1	$x + \bar{y} + \bar{z}$
1	1	1	1	1	1	1	1	1	1	

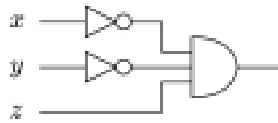
가 4 00 01 11 10 .
 - (Carnaugh map) .

1.2.1

$$\bar{x} \cdot \overline{(y + \bar{z})}$$



$$, \bar{x} \cdot \overline{(y + \bar{z})} = \bar{x}y\bar{z}$$



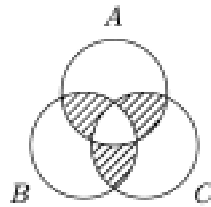
가

◇



$p(x)$ x 가 A 1, 0

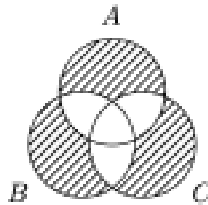
, $q(x), r(x)$ B, C x 가 1 0



x 가 1, 0 p, q, r

1.2.2

(1)

 A, B, C 

(2)



(1)

가

$$(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C)$$

(2)

가

$$(B \cup C) \cap (C \cup A) \cap (A \cup B) \cap (A^c \cup B^c \cup C^c)$$

(1)



(1)

$$(A \cup B^c) \cap (B \cup C^c) \cap (C \cup A^c)$$

(2)

$$(A \cap B^c) \cup (B \cap C^c) \cup (C \cap A^c)$$

1.2.3

(1) $F(x, y, z) = x \cdot y \cdot \bar{z} + x \cdot \bar{y} \cdot \bar{z} + \bar{x} \cdot y \cdot z + \bar{x} \cdot \bar{y} \cdot z + \bar{x} \cdot \bar{y} \cdot \bar{z}$

(2) $G(w, x, y, z) = w \cdot x \cdot y \cdot z + w \cdot x \cdot \bar{y} \cdot z + \bar{w} \cdot x \cdot y \cdot z + \bar{w} \cdot x \cdot \bar{y} \cdot z$

(1)

	00	01	11	10
0	1	1	1	0
1	1	0	0	1

	00	01	11	10
0	1	1	1	0
1	1	0	0	1

$\bar{x} \cdot z + \bar{y} \cdot \bar{z} + x \cdot \bar{z}$

$\bar{x} \cdot z + \bar{y} \cdot \bar{z} + x \cdot \bar{z}$

(2)

	00	01	11	10
00	1	0	0	0
01	0	1	1	0
11	0	1	0	0
10	0	0	0	0

	00	01	11	10
00	1	0	0	0
01	0	1	1	0
11	0	1	0	0
10	0	0	0	0

$\bar{w} \cdot \bar{x} \cdot \bar{y} \cdot \bar{z} + \bar{w} \cdot x \cdot z + x \cdot \bar{y} \cdot z$

◇

()

$(A \cap B \cap C^c) \cup (A^c \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C)$



1.2.1

$$(x + y + z)(\bar{x} + \bar{y} + \bar{z})$$



1.2.2

NOR gate NOT, OR, AND gate



1.2.3

$$(1) \overline{xy\bar{z} + \bar{x}yz + xy\bar{z}}$$

$$(2) \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + xy\bar{z}$$



1.2.4



1.2.5

:

$$(A \cap B \cap C^c) \cup (A^c \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C)$$



1.2.6

가 .
가
가

가 . 가 1, 0
, 1, 0 ,
.



1.2.7

NOT, OR, AND



1.2.8

(2002) i, j , $i j$

0, 1 가

$i \oplus j$

$$: \left\{ \begin{array}{l} 3 \rightarrow 011_{(2)} \\ 5 \rightarrow 101_{(2)} \end{array} \right\} \Rightarrow \left\{ \begin{array}{ccc} 0 & 1 & 1 \\ 1 & 0 & 1 \\ \downarrow & \downarrow & \downarrow \\ 1 & 1 & 0 \end{array} \right\} \Rightarrow 110_{(2)} \rightarrow 6 \Rightarrow 3 \oplus 5 = 6$$

, $n = 2^k - 1$
 \oplus

1.2.1

- 가 4
 . 가 5 , 6
 ?

1.2.2

NAND gate 9 XOR gate . , XOR gate
 0 , 1 gate .

1.2.3

2 2 A, B A 가 B
 , A B 가 ,
 A 가 B .
 . 가 .

2 4
 , 3 .

2.1

‘ $a \mid b$ ’는 ‘ a 가 b 를 나누다’를 의미한다.

$a, b (a \neq 0)$ 가 $b = a \cdot c$ 일 때 c 가 b 를 나누는 a 를 b 의 약수라고 한다. $a \mid b$ 라 하고, a 가 b 를 나누지 않을 때는 $a \nmid b$ 라 한다.

$a \mid b, a \nmid b$

$3 \mid 6, 3 \nmid 5, 3 \mid -6, 3 \mid 0, -2 \mid -8, 5 \nmid 7.$

1

- (1) $3 \mid 5$ 옳다.
 - (2) $3 \mid 15$ 옳다.
 - (3) $3 \mid 15$ 옳다.
- 5()3, 9()3, 3()15

- a, b, c .
- (1) $a \mid b$ $a \mid c$ $a \mid (b + c)$.
- (2) $a \mid b$ c $a \mid b \cdot c$.
- (3) $a \mid b$ $b \mid c$ $a \mid c$.

▣ (1) $a \mid b, a \mid c$ s, t $b = a \cdot s, c = a \cdot t$ 가

$$b + c = as + at = a(s + t)$$

$a \mid (b + c)$. (2), (3)

□

a d 가 q r .

$$0 \leq r < d \quad a = dq + r$$

$$, 23 = 7 \times 3 + 2 \quad (23 \quad 7$$

3, 2)

가 a, b, c 0

, a 가

2

$$-23 \quad 5$$

▣ $-23 = 5 \cdot (-5) + 2$ -5 2가 ◇

$$-23 = 5 \cdot (-4) + (-3) \quad -4, \quad \text{가 } -3$$

$$r = -3 \quad 0 \leq r < 5$$

3

- (1) $37 = 6 \cdot (\quad) + (\quad)$
 (2) $-37 = 6 \cdot (\quad) + (\quad)$
 (3) $-26 = 7 \cdot (\quad) + (\quad)$

$0 < a, b < d$. $d \mid a$ $d \mid b$ $d \mid a$
 b $\gcd(a, b)$. $a \mid c$ $b \mid c$
 $c = a \cdot b$ $\text{lcm}(a, b)$.

a, b 가 1 , $\gcd(a, b) = 1$, $a \mid b$

4

- (1) (3,5)
 (2) (15,22)
 (3) (32,18)
 (4) (35,14)

(1) $a \mid bc$ a b 가 $a \mid c$.

(2) $a \mid c, b \mid c$ a b 가 $ab \mid c$.

(1) $ab = \gcd(a, b) \cdot \text{lcm}(a, b)$. (, a, b)

(2) $\gcd(ka, kb) = k \cdot \gcd(a, b)$. (, k)

(3) .

5

A, B 가 $A \times B = 12600$ $\gcd(A, B) = 15$ $\text{lcm}(A, B)$

2.1.1

$$(2) \gcd(ka, kb) = k \cdot \gcd(a, b) \quad . \quad (\quad , k \quad)$$

$$\blacksquare \quad d = \gcd(m, n), e = \gcd(km, kn) \quad . \quad e = kd$$

$$d \mid m, d \mid n \quad . \quad kd \mid km, kd \mid kn$$

$$, kd \mid km \mid kn \quad . \quad e \quad . \quad kd \leq e \quad (\mathcal{A}).$$

$$r = \gcd(k, e), k = rk', e = re' \quad .$$

$$e \mid km, e \mid kn \iff re' \mid rk'm, re' \mid rk'n \iff e' \mid k'm, e' \mid k'n$$

$$. \quad e' \mid k' \quad . \quad (1) \quad e' \mid m, e' \mid n,$$

$$e' \mid m \mid n \quad . \quad d \quad . \quad e' \leq d, \quad e = re' \leq rd \leq kd \quad . \quad (\mathcal{A}) \quad (\quad) \quad e = kd \quad . \quad \square$$

$$\square \quad G = \gcd(A, B), L = \text{lcm}(A, B) \quad . \quad AB = GL$$

$$. \quad (1) \quad \gcd(ka, kb) = k \cdot \gcd(a, b) \quad . \quad \text{lcm}(ka, kb) = k \cdot$$

$$\text{lcm}(a, b) \quad \mathcal{A} \quad . \quad (2) \quad \text{lcm}(ka, kb) = k \cdot \text{lcm}(a, b)$$

$$\gcd(ka, kb) = k \cdot \gcd(a, b) \quad \mathcal{A} \quad .$$

2.1.2

(1) $a + b = c$, a, b, c 가 1

(2) $\gcd(a, b) = \gcd(a, a + b)$.

(1) a, b 가 d 가 $c = a + b$ d . a, c 가
 d 가 $b = c - a$ d , b, c 가 d 가
 가 . , 가
 가 . , $\gcd(a, b, c) = 1$

(2) $d = \gcd(a, b), e = \gcd(a, a + b)$. $d | a, d | b$ $d | (a + b)$.
 $d | a, a + b$, $d | e$. $e | a, e | (a + b)$
 $e | b = (a + b) - a$. $e | a, b$, $e | d$. , $d | e, e | d$
 $d = e$. □

(1) a, b, c 가 가 .
 가 1 , .
 (2) $\gcd(a, b) = 1, \gcd(a, c) = 1$ $\gcd(a, bc) = 1$.



2.1.1

$$a \mid b \quad b \mid c \quad a \mid c \quad .$$



2.1.2

$$d \mid a \quad b \mid d \quad , \quad a' \mid b' \quad d \mid dd' \quad .$$

$$d' \mid aa', ab', ba', bb'$$



2.1.3

$$\gcd(a, b) = 1 \quad \gcd(a + b, ab) = 1 \quad .$$



2.1.4

$$a \neq b \quad \text{lcm}(a + b, ab) = (a + b) \cdot \text{lcm}(a, b) \quad \text{가?}$$



2.1.5

$$a \mid b \text{가} \quad \gcd(a, c) = \gcd(a, bc) \quad .$$



2.1.6

$$(1) \quad ab = \gcd(a, b) \cdot \text{lcm}(a, b) \quad . \quad (\quad , \quad a, b \quad)$$



2.1.7

13 31

가



2.1.8

n ,
 $n/2$.
 n .

가



2.1.9

$abba$ 가 7 . $a + b$.



2.1.10

$16!$ $14!$ 가
가? , $n!$ 1 n .



2.1.11

a, b, c, d $ab - cd$. $ab - cd$
가 , $ab - cd = 1$.



2.1.12

1 : 가 600

2.1.1

가 n $n^3(n^2 - 1)(n^2 - 4)$ 가
 .

2.1.2

(1972) x, y, z
 .

$$(x, y)(x, z)(y, z)[x, y, z]^2 = [x, y][x, z][y, z](x, y, z)^2$$

, (a, \dots, g) $[a, \dots, g]$ $\gcd(a, \dots, g)$ $\text{lcm}(a, \dots, g)$
 .

2.1.3

(1980 Putnam) r s .

$$3^r 7^s = \text{lcm}(a, b, c) = \text{lcm}(a, b, d) = \text{lcm}(a, c, d) = \text{lcm}(b, c, d)$$

(a, b, c, d) r s
 .

2.2

가 1 . 1
 . , 가 1 1 ,
 2 가 , 3 가 .

1

$9 = 3 \cdot 3, 18 = 3 \cdot 3 \cdot 2 = 3^2 \cdot 2$, 2, 5, 13 .

‘ ’ ‘ (素)’ ‘ ’ ,

. ,

. 가 , ()

.

. 가

$$m \sqrt{m} \qquad m$$

◻ : ‘ m 가 ’, m
 . $m - 1$ a . , $m = ab$,
 $1 < a, b < m$. 가 $m \sqrt{m}$ 가 $a, b > \sqrt{m}$

$$m = ab > \sqrt{m}\sqrt{m} = m$$

. , m 가 . m . ◻

2

127

$11 < \sqrt{127} < 12$. 127 11
 . 127 2, 3, 5, 7, 11 가 □

3

가

- (1) 217
- (2) 297
- (3) 299
- (4) 319

$$, 540 = 2^2 \times 3^3 \times 5$$

4

- (1) 400
- (2) 378
- (3) 736
- (4) 1260

가
 . 1
 . 1 , 10

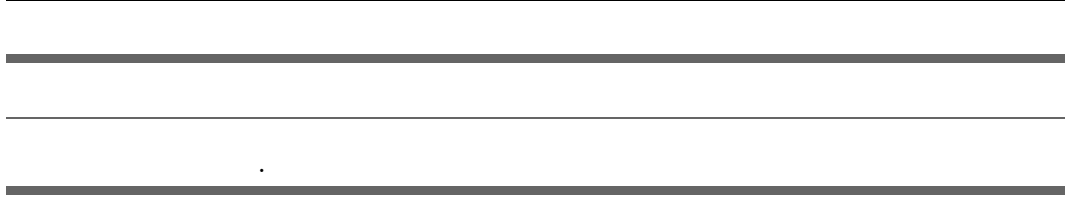
$$10 = 2 \times 5 = 1 \times 2 \times 5 = 1 \times 1 \times 2 \times 5 = \dots$$

가 가
 가 ‘ ’ , 가 가
 . 가 가 ,
 가 가
 , , 가
 ,
 .

가 가 .

가 , 가

$x^n = a$ 가 $x = a^{1/n}$ 가
 . 2 8 3 , 3 81 .



: 가 가 .

$$p_1, p_2, \dots, p_n \quad (p_1 < p_2 < \dots < p_n)$$

. A 1 . ,

$$A = p_1 p_2 \cdots p_n + 1$$

. A p_1, p_2, \dots, p_n 1 A

. A . $A = p_1 p_2 \cdots p_n + 1 >$

p_n A p_1, p_2, \dots, p_n .

p_1, p_2, \dots, p_n 가 . ,

□

2.2.1

$\sqrt{2}$ 가

, $\sqrt{2}$ 가

$\sqrt{2} = \frac{r}{s}$ 가

r, s 가

$$2s^2 = r^2$$

, r

$r = 2t$

$$s^2 = 2t^2$$

, s

r, s

, r, s 가

(2)

$\sqrt{2}$ 가

가

, $\sqrt{2}$

□

$\sqrt[3]{3}$

2.2.2

5

가

.

5 $n - 2, n - 1, n, n + 1, n + 2$. ($n \geq 3$)

$$(n - 2)^2 + (n - 1)^2 + n^2 + (n + 1)^2 + (n + 2)^2 = 5(n^2 + 2)$$

$n^2 - 5$ 0, 1, 4 가 , $n^2 + 2$
 2, 3, 1 . , $n^2 + 2$ 5 가 . $5(n^2 + 2)$
 5가 , 가 . \square

 n

$$n^4 - 20n^2 + 4$$

가

.

2.2.3

(1) n $10 < n < 30$, $\sqrt{3n}$ 가 n .
 (2) .

(1) $3n$ $n = 3 \times (\quad)$. 1, 4, 9, 16, ... 3 $10 < n < 30$ $n = 12$ 27.
 n $12 + 27 = 39$.

(2) 1 . 1 n^2 $n = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}$
 . $(p_1, p_2, \dots, p_n$, a_1, a_2, \dots, a_n)

$$n^2 = p_1^{2a_1} p_2^{2a_2} \cdots p_n^{2a_n}$$

, n^2

$$(2a_1 + 1)(2a_2 + 1) \cdots (2a_n + 1)$$

◇

(2) n . $a \mid n$
 $\frac{n}{a} = b \mid n$ $b \mid n$ $\frac{n}{b} = a \mid n$, $ab = n$ (a, b) 가
 . a b 가 n 가 ,
 $a = b$ 가 $n = a^2$ n 가

□

$10 < \sqrt{6a} < 30$, $\sqrt{6a}$ 가 a

2.2.4

(1981 가) $2^8 + 2^{11} + 2^n$ 가
 n .

$$\blacksquare \quad 2^8 + 2^{11} = 48^2 \quad . \quad 2^8 + 2^{11} + 2^n = k^2$$

$$k^2 - 48^2 = (k + 48)(k - 48) = 2^n$$

, $(k + 48)$ $(k - 48)$ 2 . , 2
 가 96 가 가 .

1, 2, 4, 8, 16, 32, 64, 128, 256, ...

가 96 $(128, 32)$. $2^n = 128 \cdot 32 = 2^{12}$,
 $(2^4 + 2^6)^2 = 80^2$. n

□

$$\square \quad 3^{14} + 3^{13} - 12 \quad \text{가} \quad .$$



2.2.1

가

가 3

.



2.2.2

가?



2.2.3

 $\frac{n}{2}$ $, \frac{n}{3}$
 n $, \frac{n}{5}$
.

가

가



2.2.4

 $\sqrt[3]{10}$

.



2.2.5

 $2n + 1, 3n + 1$ $, n$

.

 $5n + 3$

가

.



2.2.6

n) $11 \cdots 1211 \cdots 1($ 1 .



2.2.7

$2^n - 1$ n .



2.2.8

19 1 11111111111111111111 .
 b , b $11 \cdots 1_{(b)}$ 가 가 1 가



2.2.9

p q 1 가
 . $p > 100$, $p + q$ 가
 .



2.2.10

p 가 p 30 . ,
 1 .



2.2.11

$7^{12} + 4^{12}$ 가 2626 , 가 .



2.2.12

$y = ax^2 + 19x$ 가 y - 가 . , a , x - y - 가 .



2.2.13

(1960 Putnam) n .
 $\frac{xy}{x+y} = n$
 (x, y) 가?



2.2.14

(1967 Putnam) 1 50 50
 , 1 50 가 가
 . , 1
 가 , 2 가
 . , 3
 가 3 ,
 가, 가
 , ?

2.2.1

(1947 Putnam) a, b, c, d ,

$$(x - a)(x - b)(x - c)(x - d) - 4 = 0$$

$$r \quad . \quad 4r = a + b + c + d \quad .$$

2.2.2

2000 3 .

$$\cdot \quad 7497 \quad 4284 \quad \cdot$$

$$7497 = 4284 \cdot 1 + 3213$$

$$\cdot, \gcd(4284, 3213) \quad \cdot$$

$$4284 = 3213 \cdot 1 + 1071$$

$$\cdot, \gcd(3213, 1071) \quad \cdot,$$

$$3213 = 1071 \cdot 3$$

$$\cdot, \quad 1071 \quad \cdot$$

.

7497	19278
	$7497 \times 2 = 14994$
7497	4284
4284	
3213	4284
	3213
3213	1071
$1071 \times 3 = 3213$	
0	

- (1) (105,490)
- (2) (1254,540)
- (3) (2146,2491)
- (4) (12574,33120)

$$\gcd(a, b) = \gcd(b, r) \quad ?$$

$$a = bq + r \qquad \gcd(a, b) = \gcd(b, r) \quad .$$

□ q 가 , $a = bq + r \implies a \equiv r \pmod{b}$
 . , $\gcd(a, b) = m, \gcd(b, r) = n$.
 $m \mid a, m \mid b$ (\because) , $m \mid (a - bq), \implies m \mid r$
 . , $m \mid b, m \mid r$ 가 , $n \mid b, n \mid r$.
 $\therefore m \leq n$.
 $a \equiv r \pmod{b} \implies a - r \equiv 0 \pmod{b}$, $n \mid a - r$ 가 $n \mid a - r$
 가 , $n \leq m$.
 $\therefore m = n$. □

□ $a = bq + r$,
 가 (, $0 \leq r < b$ 가) . ,
 $a = bq + r$
 . r

가 . 가
 , 가
 .

2.3.1

x 가

$$\frac{x^2 + x + 1}{x^3 + 3x^2 + 4x + 3}$$



가

가 1

가



가

가

가

$$\begin{array}{r|l}
 \begin{array}{r}
 x^3 + 3x^2 + 4x + 3 \\
 x^3 + 3x^2 + 3x + 2 \\
 \hline
 x + 1 \\
 \hline
 \end{array}
 &
 \begin{array}{l}
 x^2 + x + 1 \\
 x^2 + x + 1 \\
 x^2 + x \\
 \hline
 1
 \end{array}
 \end{array}
 \quad x + 2$$

$$x^2 + x + 1$$

$$x^3 + 3x^2 + 4x + 3$$

$$1$$

x



(1) $\frac{x^3}{x^3 + x - 1}$

(2) $\frac{x^3 + x^2 - x + 1}{2x^3 + 2x^2 - 2x + 1}$

2.3.2

880, 1238, 2133 $d (d > 0)$ 가 r
 $, d - r$.

d 가 r , d

$$d \mid (1238 - 880) = 358, \quad d \mid (2133 - 1238) = 895$$

d 358 895 . 358 895 ,

2	358	895	
		716	
	358	179	2
	358		
	0	179	

179 , d 179 . 179 , d 1

179 .

(i) $d = 179$; 880, 1238, 2133 179 가 164
 . , $d - r = 179 - 164 = 15$.

(ii) $d = 1$; 880, 1238, 2133 1
 0 . , $d - r = 1 - 0 = 1$.

(i), (ii) , $d - r$ 1 15 . ◇

(1988 KMO) $a = 11, 111, 111$ $b = 11, 111, \dots, 111$ (1
 1988) .



2.3.1

(1) (1276,3212)

(2) (1362,780)



2.3.2

(1) (2725,375)

(2) (833,9905)



2.3.3

(1959 IMO)

 n

$$\frac{21n + 4}{14n + 3}$$



2.3.4

 $4n + 3$ $7n + 4$

가 1

 n 

2.3.5

1



2.3.6

$$(n+1)(3n+1) \mid (2n+1)^2 \quad .$$



2.3.7

$$k \mid n, \gcd(n, n+k) = 1 \quad n > 1 \quad n \\ , k = 1 \quad .$$



2.3.8

 n
 $.$


2.3.9

$$ad - bc = 1 \quad \frac{a+b}{c+d} \notin \mathcal{Z} \quad .$$



2.3.10

$$2+1, 2^2+1, 2^4+1, 2^8+1, \dots, 2^{2^n}+1, \dots$$

2.3.1

n a_n ,

$$\frac{a_n}{a_{n+1}}$$

.

, $a_0 = a_1 = 1, a_{n+1} = a_n + a_{n-1} (n \geq 1)$

.

2.3.2

(1956 Putnam) $T_1 = 2, T_{n+1} = T_n^2 - T_n + 1$

$(n > 0)$. $n \neq m$ $T_n T_m$

.

3.1

가 , , 가 , 가
가 .

■

가 .

1

$$\begin{aligned}x - 2y + z &= 0 \\2y - 8z &= 8 \\-4x + 5y + 9z &= -9\end{aligned}$$

가 ,

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix}$$

$$\xrightarrow{(A)} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{bmatrix} \xrightarrow{(B)} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & -9 \end{bmatrix}$$

(A): () 4 () .

(B): 가 $\frac{1}{2}$.

$$\xrightarrow{(C)} \begin{bmatrix} 1 & 0 & -7 & 8 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{(D)} \begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

(C): 가 2 , 3 , .

(D): 7 , 4 , 가 .

$$1 \cdot x + 0 \cdot y + 0 \cdot z = x = 29$$

$$0 \cdot x + 1 \cdot y + 0 \cdot z = y = 16$$

$$0 \cdot x + 0 \cdot y + 1 \cdot z = z = 3$$

$$, (x, y, z) = (29, 16, 3) .$$

$$1, 0$$

가 .

2

$$y + 5z = -4$$

$$x + 4y + 3z = -2$$

$$2x + 7y + z = -1$$

... ■ 가 . (2

0 = -1 . 가

.)

가

$$3b - 6c + 6d + 4e = -5$$

$$3a - 7b + 8c - 5d + 8e = 9$$

$$3a - 9b + 12c - 9d + 6e = 15$$

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \xrightarrow{(A)} \begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

$$(A): 1 \quad 3 \quad .$$

$$\xrightarrow{(B)} \begin{bmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix} \xrightarrow{(C)} \begin{bmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

$$(B): 1 \quad \frac{1}{3} \quad .$$

$$(C): 1 \quad 3 \quad 2 \quad .$$

$$\xrightarrow{(D)} \begin{bmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix} \xrightarrow{(E)} \begin{bmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$(D): 2 \quad \frac{1}{2} \quad .$$

$$(E): 2 \quad 3 \quad 3 \quad .$$

$$a - 3b + 4c - 3d + 2e = 5$$

$$b - 2c + 2d + e = -3$$

$$e = 4$$

$$c \quad d \quad c = t, d = u \quad (\quad) \quad ,$$

$$e = 4$$

$$d = u$$

$$c = t$$

$$b = 2c - 2d - e - 3 = 2t - 2u - 7$$

$$a = 3b - 4c + 3d - 2e + 5 = 2t - 3u - 24$$

,

$$\begin{bmatrix} \bullet & * & * & * & * & * \\ 0 & \bullet & * & * & * & * \\ 0 & 0 & 0 & 0 & \bullet & * \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & * & * & 0 & * \\ 0 & 1 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$

• 0

, *

.

•

1

.

,

가

.

4

$$x + 4y + 2z = 3$$

$$-2x - 8y - z = 9$$

$$\dots \blacksquare (x, y, z) = (-4t - 7, t, 5) \quad (t \in \mathbb{Z})$$

■ 가

가

가

.

5

1000 m . 50 가 30
 . 50 가 90
 .

 $x m$, $y m$. 가
 가

$$50(x - y) = 30(x + y)$$


. 가 가

$$1000 + 50(x + y) = 90(x - y)$$

가 . $20x = 80y, 40x = 140y + 1000$
 가 , $x = 4y, 2x = 7y + 50$.
 $y = 50$. , $50 m$. ◇

6

1 ha 20 ,
 15 .
 , ?

 . 3 20
 , 5 12 () . , 2
 15 , 5 6 . , 18
 . ◇

7

$$y = |x + 1| + |x - 3|, y - x = 2$$

(1) $x < -1$; $y = -(x+1) - (x-3) = -2x+2$ 가 ,
 $(x, y) = (0, 2)$ 가 , x

(2) $-1 \leq x < 3$; $y = (x+1) - (x-3) = 4$ 가 ,
 $(x, y) = (2, 4)$.

(3) $3 \geq x$; $y = (x+1) + (x-3) = 2x-2$ 가 ,
 $(x, y) = (4, 6)$.

(1)-(3) (2, 4) (4, 6) . \diamond

8

a, b, c

$$ab = 2(a+b), \quad bc = 3(b+c) \quad ca = 4(c+a)$$

$$5a + 7b + c \quad ?$$

$$2ab \quad \frac{1}{2} = \frac{a+b}{ab} = \frac{1}{a} + \frac{1}{b} \text{ 가 } .$$

$$\frac{1}{2} = \frac{1}{a} + \frac{1}{b}, \quad \frac{1}{3} = \frac{1}{b} + \frac{1}{c}, \quad \frac{1}{4} = \frac{1}{c} + \frac{1}{a}$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 2 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \quad , \quad \frac{13}{24} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

$$\frac{1}{c} = \frac{1}{24}, \quad \frac{1}{a} = \frac{5}{24}, \quad \frac{1}{b} = \frac{7}{24} \quad , \quad 5a = 7b = c = 24$$

$$, 5a + 7b + c = 72 \quad . \quad \diamond$$

3.1.1

100

$$\begin{aligned}
 & \cdot \\
 x_1 + x_2 + x_3 &= 0 \\
 x_2 + x_3 + x_4 &= 0 \\
 & \vdots \\
 x_{98} + x_{99} + x_{100} &= 0 \\
 x_{99} + x_{100} + x_1 &= 0 \\
 x_{100} + x_1 + x_2 &= 0
 \end{aligned}$$



100

x_i 가

$$3(x_1 + x_2 + x_3 + \cdots + x_{99} + x_{100}) = 0$$

$$, x_1 + x_2 + \cdots + x_{100} = 0 \quad . \quad \begin{matrix} 100 \\ 0 \end{matrix} \quad 33 \quad .$$

$$0 = x_1 + (x_2 + x_3 + x_4) + (x_5 + x_6 + x_7) + \cdots + (x_{98} + x_{99} + x_{100}) = x_1$$

$$\begin{aligned}
 & \cdot \quad x_i \quad (x_{i+1} + x_{i+2} + x_{i+3}), \dots \\
 & \quad (x_{i-3} + x_{i-2} + x_{i-1}) \quad 33
 \end{aligned}$$

$$, x_i = 0 \quad . \quad , x_1 = x_2 = \cdots = x_{100} = 0. \quad \diamond$$



?

100

가



a, b, c, d, e

$$ab = 1, \quad bc = 2, \quad cd = 3, \quad de = 4, \quad ea = 6$$

3.1.2

(1987) a, b, c 가 0

$$\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a}$$

$a+b+c=0$ $a=b=c$.

$\frac{p}{q} = \frac{r}{s}$, $q+s$ 가 0 $\frac{p+r}{q+s}$ 가 . 가

$a+b+c=0$. (,
 -2가 .) , $a+b+c \neq 0$.

$$\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a} = k \tag{*}$$

, 가

$$k = \frac{(a+b-c) + (a-b+c) + (-a+b+c)}{a+b+c} = \frac{a+b+c}{a+b+c} = 1$$

. , (*)

$$a+b-c=c, \quad a-b+c=b, \quad -a+b+c=a$$

. $2c, 2b, 2a$

$$a+b+c = 3a = 3b = 3c$$

. , $a=b=c$. (, $k=1$.)

$\frac{a}{b+c+d} = \frac{b}{c+d+a} = \frac{c}{d+a+b} = \frac{d}{a+b+c}$,

3.1.1

$$(1) \begin{cases} x - 6y & = 5 \\ y - 4z + w & = 0 \\ -x + 6y + z + 5w & = 3 \\ -y + 5z + 4w & = 0 \end{cases}$$

$$(2) \begin{cases} x - 2y - z + 3w & = 0 \\ -2x + 4y + 5z - 5w & = 3 \\ 3x - 6y - 6z + 8w & = 2 \end{cases}$$

 x, y

3.1.2

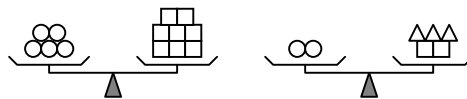
$$x + hy = 1$$

$$2x + 3y = k$$

(1) h, k (2) 가 h, k (3) 가 h, k

3.1.3

○, □, △가



○ 3

△

가?



3.1.4

$$f(t) = a + bt + ct^2 \quad (1, 3), (2, 5), (3, 6)$$

. a, b, c .



3.1.5

$$2x - y = 6z, -5x + 4y = 3z \text{ 가 } ,$$

(1) x, y z .

(2) x, y, z 가 504 , x, y, z .



3.1.6

55 . 55 $\frac{1}{5}$ 3 ,

2 .

5 . .

(1) .

(2) .



3.1.7

가 A, B, C 4
 , A, C 6 , B, C 6 40
 . A, B, C 가?



3.1.8

(2002) x, y, z 가
 $x^2 + y^2 + z^2 = 3$, $xyz \neq 0$.

$$(1) x + y + z = 3$$

$$(2) x^2 \left(\frac{1}{y} + \frac{1}{z} \right) + y^2 \left(\frac{1}{z} + \frac{1}{x} \right) + z^2 \left(\frac{1}{x} + \frac{1}{y} \right) = -3$$



3.1.9

x, y, z ,

$$x^2 - kyz = y^2 - kzx = z^2 - kxy$$

, k $x + y + z$.



3.1.10

a, b, c ,

$$a(1 - b) = b(1 - c) = c(1 - a) = k$$

, k .



3.1.11

(1896 가 Eötvös) $x^2 - 3xy + 2y^2 + x - y = 0$ $x^2 - 2xy + y^2 - 5x + 7y = 0$ $x^2 - 3xy + 2y^2 + x - xy - 12x + 15y = 0$

· ,

·



3.1.12

(2003)

·

$$x^3 + y^3 + z^3 = x + y + z$$

$$x^2 + y^2 + z^2 = xyz$$

3.1.1

(1977 Putnam)

 x, y, z, w .

$$x + y + z = w, \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{w}$$

3.1.2

 a, b, c 가 , x, y, z .

$$x^2 - yz = a^2$$

$$y^2 - zx = b^2$$

$$z^2 - xy = c^2$$

3.1.3

 a, b, c, d, e, f 가

$$a + b + c + d + e + f = 0$$

$$a^3 + b^3 + c^3 + d^3 + e^3 + f^3 = 0$$

,

$$(a + c)(a + d)(a + e)(a + f) = (b + c)(b + d)(b + e)(b + f)$$

.

, a 가 t . , t 가
 , b $10 - 2t$ 가 3 가 .
 $10 - 2t =_3 t + 1 =_3 0$, $t = 3k + 2$. ,
 $a = 3k + 2, b = -2k + 2$ (k)

$(a, b) = (3k - 1, 4 - 2k)$ $(a, b) = (2 - 3k, 2 + 2k)$

, . \diamond

2

0 $2x + 3y = 0$ 가 1
 . (X, Y)
 , (x_1, y_1) ,
 $(x, y) = (x_1, y_1) + (X, Y)$.

$2x + 3y = 10$
 $2x_1 + 3y_1 = 10$

,
 $2(x - x_1) + 3(y - y_1) = 0$
 , $(x - x_1, y - y_1) = (X, Y)$ 가 .
 . $(x_1, y_1) = (2, 2)$
 . $2x + 3y = 0$
 3 $2x$ 가 3 , x 가 3 가
 , $x = 3k$ $y = -2k$ 가 .
 $(x, y) = (3k, -2k) + (2, 2)$. \diamond

2

$2a + 3b + c = 10$ (a, b, c) .

a, b 가 $c = 10 - 2a - 3b$. , $2a + 3b < 10$
 (a, b) . $3b \geq 3$ $a = 1, 2, 3$ 가 .

$$(a, b, c) = (1, 1, 5), (1, 2, 2), (2, 1, 3), (3, 1, 1)$$

◇

■ 가

가

가

가

3

$$3xy - 7y^2 + 2x - 5y + 15 = 0$$

 가 가 x .

$$x = \frac{7y^2 + 5y - 15}{3y + 2} = \frac{7}{3}y + \frac{1}{9} - \frac{\frac{137}{9}}{3y + 2}$$

$$9x = 21y + 1 - \frac{137}{3y + 2}$$

$9x = 21y + 1$, 가 $3y + 2$
 가 137 . 137 ,

$$3y + 2 = 1, -1, 137, -137$$

가 . y , 가 $y = -1, 45$.
 x $(x, y) = (13, -1), (105, 45)$ 가

◇

4

$$x^2 + xy - 2y^2 - 7 = 0$$

$$(x + 2y)(x - y) = 7$$

$$x + 2y = 7, \quad x - y = 1$$

가

$$(x + 2y, x - y) = (1, 7), (7, 1), (-1, -7), (-7, -1)$$

$$3y = \pm 6 \quad y = \pm 2$$

$$(x, y) = (3, 2), (-5, 2), (5, -2), (-3, -2)$$

◇

5

$$x^2 + y^2 - 12x + 2 = 0$$

$$x^2 - 12x + (y^2 + 2) = 0 \quad (x, y) \text{가}$$

x 가 , y 가

$$D = (-12)^2 - 4(y^2 + 2) \geq 0$$

$$\text{가} \quad , |y| \leq \sqrt{34},$$

$$y = 1, 2, 3, 4, 5$$

$$x^2 - 12x + 3 = 0$$

$$x^2 - 12x + 6 = 0$$

$$x^2 - 12x + 11 = 0$$

$$x^2 - 12x + 18 = 0$$

$$x^2 - 12x + 27 = 0$$

5 . 가 ,
 $(x, y) = (1, 3)$ $(11, 3)$, $(x, y) = (3, 5)$
 $(9, 5)$. ◇

2

$$(x - 6)^2 + y^2 = 34$$

. 34 0, 1, 4, 9, 16, 25 ,
 34가 25 + 9 . ,

$$(|x - 6|, y) = (5, 3) \quad (|x - 6|, y) = (3, 5)$$

, . ◇

6

$$y^2 + 2002 = x^2$$

$$(x + y)(x - y) = 2002 \quad . \quad x + y \quad x - y$$

. 2002가
 . 4 가 .
 2002 4 가 . ,

◇

○

$$4 \quad \text{가} \quad 0 \quad 1$$

가 1

,

.

7

A 5 970 , 3 670 ,
 250 . A 100 가
 . A 20,000 A 가
 가?

5 x , 3 y , z .
 100 , 20,000

$$5x + 3y + z = 100, \quad 970x + 670y + 250z = 20000$$

$$z = 100 - 5x - 3y$$

$$7x + 2y = 125$$

가 . $x = 2k + 1$
 . $y = 59 - 7k, \quad z = 100 - 5x - 3y$
 $z = 11k - 82$. x, y, z 가

$$2k + 1 \geq 0, \quad 59 - 7k \geq 0, \quad 11k - 82 \geq 0$$

, k 가

$$k \geq 0, \quad k \leq 8, \quad k \geq 8$$

, $k = 8$. , $z = 6$
 . ◇

□ x, y, z z 가
 . $x = y$, 가
 ?

8

1993 10 21 , 가
 () . 가 ? , 10 21

가 $abcd$, $1000a + 100b + 10c + d$

$$1993 - (1000a + 100b + 10c + d) = a + b + c + d$$

$$1001a + 101b + 11c + 2d = 1993$$

a, b, c, d 0 9

가 a 가 가

$$0 \leq 101b + 11c + 2d \leq 909 + 99 + 18 = 1026$$

$$1001a \leq 1001a + 101b + 11c + 2d = 1993 \leq 1001a + 1026$$

$$, 967 \leq 1001a \leq 1993 , a = 1$$

$$101b + 11c + 2d = 992$$

가 , $11c + 2d$ 가 0 117 101b
 875 992 , $b = 9$.

$$11c + 2d = 83$$

$2d$ 0 18 $11c$ 65 83
 , c 6 7 . c 가 6 d 가 가 .
 $c = 7, d = 3$ 가 , 1973 . \diamond

(m, n) .

$$n^2 + (n + 1)^2 = m^4 + (m + 1)^4$$



, $0 + 1$ 가
 . 가



$n - m$, $n^2 + (n + 1)^2$
 $m^4 + (m + 1)^4$
 . $n - (n + 1)$, $m - (m + 1)$
 , m, n

$n(n + 1) = (m^2 + m)(m^2 + m + 2)$
 , $l = m^2 + m$,

$$n(n + 1) = l(l + 2)$$

,
 가 . , $3 \cdot 4$
 $l(l + 2)$, $2 \cdot 4$ $3 \cdot 5$

$n \leq l$, n, l 가 ,
 $n \leq l$. $n \leq l$

$$n(n + 1) \leq l(l + 1) \leq l(l + 2)$$

$l \neq 0$. $l \neq 0$ $n > l$
 . $n \geq l + 1$

$$n(n + 1) \geq (l + 1)(l + 2) > l(l + 2)$$

. $n < l + 1$.
 가 $n = l = 0$, . , n, l

, m, n $0 - 1$ 가
 가 . \diamond

 m n

가

3.2.1

$$5x - 6y = 11$$

[1]

가

$$x = \frac{6y + 11}{5} = y + 2 + \frac{y + 1}{5}$$

$\frac{y + 1}{5}$ 가 y 가

$$y = \dots, -6, -1, 4, 9, 14, \dots$$

[2] x, y 가

$$y > 0$$

$$x = y + 2 + \frac{y + 1}{5} > 2 + \frac{1}{5} > 0$$

$$x > 0$$

$$y > 0$$

$$y = 4, 9, 14, \dots$$

[3] ,

y

x

(x, y)

$$(7, 4), (13, 9), (19, 14), \dots$$

◇

$$7x - 4y = 13$$

3.2.2

$$2a + 3b + 5c = 10$$

$$\begin{aligned}
 & \text{5c} & 2a + 3b = 5(2 - c), & & 5 \\
 & 5 & , & & 5 & c \\
 & & . & , & 2a + 3b = 5k & , c = 2 - k \\
 & & & & . & \\
 & & . & 2a + 3b = 5k & & (a, b) = (k, k) \\
 & & & . & & 2A + 3B = 0 \\
 & (A, B) = (3t, -2t) & . & 2a + 3b = 5k & & \\
 & & & & & (a, b) = (3t, -2t) + (k, k) = (3t + k, -2t + k)
 \end{aligned}$$

가 ,

$$(a, b, c) = (3t + k, -2t + k, 2 - k) \quad (t, k)$$

가 .

◇

$$\begin{aligned}
 & a + 2b + 3c = 5, \quad 2a - b + 4c = 1 \\
 & (a, b, c) .
 \end{aligned}$$

3.2.3

가 가



가 $c, a, b (a \geq b)$.

$$a + b + c = \frac{ab}{2}, \quad c = \frac{ab}{2} - (a + b)$$

. $a^2 + b^2 = c^2$, c

$$a^2 + b^2 = \frac{a^2b^2}{4} - ab(a + b) + a^2 + b^2 + 2ab$$

$$ab(a + b) = \frac{a^2b^2}{4} + 2ab$$

$$a + b = \frac{ab}{4} + 2$$

$$ab - 4a - 4b + 8 = 0$$

$$(a - 4)(b - 4) = 8$$

. 가 8 $a \geq b$

$$(a - 4, b - 4) = (8, 1), (4, 2), (-2, -4), (-1, -8)$$

가 , $a, b (a, b) = (12, 5) (8, 6)$

.

$$\{a, b, c\} = \{5, 12, 13\} \quad \{6, 8, 10\}$$

. ◇



가 가

. (, .)

3.2.4

a, b, c

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{4}{5}$$

$\frac{1}{n} \leq 1$

$a \leq b \leq c$

$$\frac{4}{5} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{1}{a} + \frac{1}{a} + \frac{1}{a} = \frac{3}{a}$$

$a \leq \frac{15}{4}$, $\frac{1}{a} \leq \frac{4}{5}$, $a \geq \frac{5}{4}$, a 2

3

(1) $a = 2$, $\frac{1}{b} + \frac{1}{c} = \frac{3}{10}$, $\frac{10}{3} \leq$

$b \leq \frac{20}{3}$, $b = 4, 5, 6$, $(a, b, c) =$

$(2, 4, 20)$ $(2, 5, 10)$, $b = 6$, $c = \frac{15}{2}$ 가

(2) $a = 3$, $\frac{1}{b} + \frac{1}{c} = \frac{7}{15}$, c 가 $\frac{15}{2}, \frac{60}{13}$

$\frac{15}{7} \leq b \leq \frac{30}{7}$, $b = 3, 4$, c 가 $\frac{15}{2}, \frac{60}{13}$

가

, (1) (2) $(2, 4, 20)$ $(2, 5, 10)$,

◇





3.2.1

(1990 . . .)
 $33x + 23y = 1$. . . (x, y) . . .



3.2.2

(1) $3x + 5y = 28$
(2) $5x + 7y = 94$
(3) $14x + 21y = 149$



3.2.3

$2x + 3y + 7z = 23$. . .



3.2.4

. . . (x, y, z)
 $x + y + z = 10, \quad 2(x + y) = z$



3.2.5

(1) $xy - 2x - 2y + 7 = 0$
(2) $3x^2 - xy = 10$
(3) $x^2 - y^2 = 117$



3.2.6

$2xy + 3x - 5y = 12$. . . (x, y) . . .



3.2.7

5 . ‘ ’ 2 5
 ‘ ’ 68 .
 가?



3.2.8

가 350 .
 10 . ? , 10 ,
 50 , 100 가 .



3.2.9

$x, y, z \geq 3$, $\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \frac{1}{2}$.



3.2.10

$2x^2y^2 + y^2 = 26x^2 + 1201$ (x, y)



3.2.11

$\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = 1$.



3.2.12

21 가 241 .



3.2.13

가



3.2.14

$$x > y > 0, \quad x^3 + 7y = y^3 + 7x$$



3.2.15

$$4x + y = 3xy$$



3.2.16

$$x, y, z, \quad z, \quad x(x + y) = z + 120$$



3.2.17

$$x^2 + y^2 + z^2 = 1999$$



3.2.18

$$1987 \quad b \quad xyz \quad x + y + z = 25$$

$$, x, y, z \quad b$$

3.2.1

:

 x, y, z

$$x - y + z = 1 \quad \text{가}$$

,

.

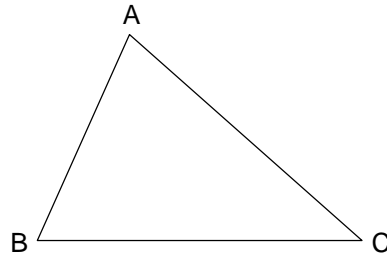
3.2.2

 (x, y) .

$$x^3 + x^2y + xy^2 + y^3 = 8(x^2 + xy + y^2 + 1)$$

3.3

$\overline{AB} + \overline{BC} > \overline{AC}$ 가 . A C AC
 가 가 .



가 . , , 가
 가 .
 () \geq ()

1

a, b, c 가

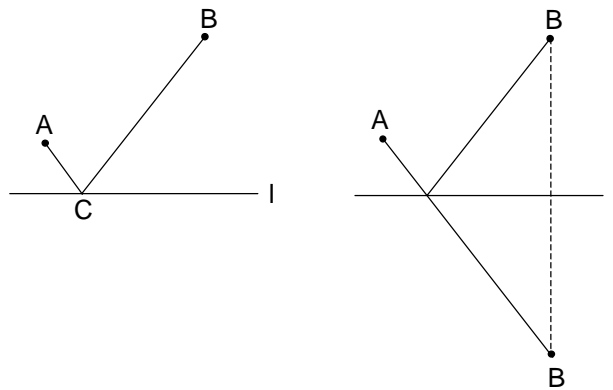
$$a < b + c, \quad b < c + a, \quad c < a + b$$

c AB , A, B b, a
 가?

가 가 , 가 .

2

()
 A 가 B 가
 ℓ C A C , C B
 $\overline{AC} + \overline{CB}$ 가
 C 가?



$\overline{CB} = \overline{CB'}$.

$$\overline{AC} + \overline{CB} = \overline{AC} + \overline{CB'}$$

. , A, C, B 가 , A B'
 가 A, C, B'
 . C $\overline{AB'}$ ℓ . \diamond

3

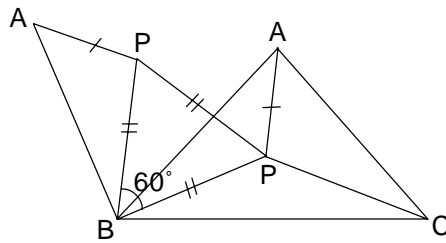
()
 A B . A B
 . A B
 가?

$$\sqrt{\left(1 + \frac{1}{2}\right)^2 + \left(1 + 1 + \frac{1}{2}\right)^2} = \frac{\sqrt{34}}{2}$$

◇

5

()
 $\triangle ABC$ $\overline{PA} + \overline{PB} + \overline{PC}$ P



$\triangle P'BP$ $\triangle A'P'B$ $\triangle APB \cong \triangle A'P'B$ $\angle PBP' = 60^\circ$ $\overline{PB} = \overline{P'B}$ $\overline{AB} = \overline{A'B}$ $\angle PBA = \angle P'BA'$

$$\overline{PA} + \overline{PB} + \overline{PC} = \overline{A'P'} + \overline{P'P} + \overline{PC} \geq \overline{A'C}$$

$$\begin{aligned} \angle BPC &= 180^\circ - \angle BPP' = 180^\circ - 60^\circ = 120^\circ \\ \angle APB &= \angle A'P'B = 180^\circ - \angle BP'P = 180^\circ - 60^\circ = 120^\circ \\ \angle CPA &= 360^\circ - (\angle BPC + \angle APB) = 120^\circ \end{aligned}$$

$\angle APB = \angle BPC = \angle CPA = 120^\circ$ 가

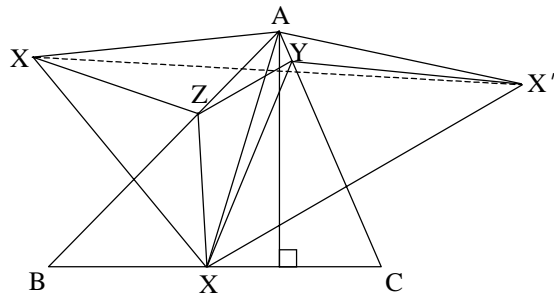
◇

□ Fermat

6

ABC

(, , .)



\overline{BC} X , \overline{AC} Y , \overline{AB} Z
 $\triangle ABC$ XYZ .
 \overline{AB} X X' , \overline{AC} X X'' .

$$\overline{XY} + \overline{YZ} + \overline{ZX} = \overline{X''Y} + \overline{YZ} + \overline{ZX'} \geq \overline{X'X''}$$

, X , $X'ZYX''$ 가 $\overline{XY} + \overline{YZ} + \overline{ZX}$.

, $\overline{AX'} = \overline{AX} = \overline{AX''}$. $\triangle X'AX''$, $\angle X'AX''$

(2 $\angle A$) $\overline{X'X''}$ \overline{AX} 가 .

$\overline{AX} \perp \overline{BC}$.

가 $\overline{BY} \perp \overline{AC}$, $\overline{CZ} \perp \overline{AB}$. , $\triangle XYZ$

가 . \diamond

x, y 가

$$|x| + |y| \geq |x + y|$$

x, y 가



$$(|x| + |y|)^2 \geq |x + y|^2$$

$$|x|^2 + 2|x||y| + |y|^2 \geq (x + y)^2$$

$$x^2 + 2|xy| + y^2 \geq x^2 + 2xy + y^2$$

$$|xy| \geq xy$$

xy 가 0, $x,$

y 가

□



$|x|$ x

, (0) x x 가

2 3

7

a, b, c, d ,

$$|a + b + c| \leq |a - 2d| + |b + d| + |c + d|$$

가



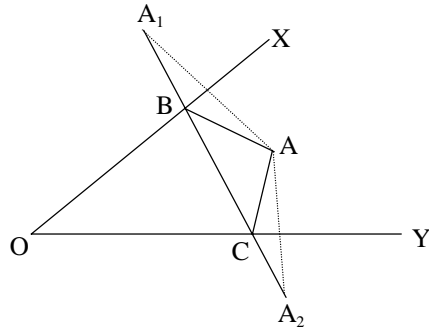
$$\begin{aligned} |a + b + c| &= |(a - 2d) + (b + c + 2d)| \\ &\leq |a - 2d| + |b + c + 2d| \\ &\leq |a - 2d| + |b + d| + |c + d| \end{aligned}$$

□

3.3.1

$\angle XOY$ A 가 . , $OX,$ OY
 B, C $\triangle ABC$, $가$ $가$ $B,$
 C .

$A_1,$
 A_2 .



$$\overline{AB} = \overline{A_1B} \quad \overline{AC} = \overline{A_2C} \quad .$$

$$\overline{AB} + \overline{BC} + \overline{CA} = \overline{A_1B} + \overline{BC} + \overline{CA_2}$$

. A_1, A_2 $\overline{A_1A_2}$ $가$

$$\overline{A_1B} + \overline{BC} + \overline{CA_2} \geq \overline{A_1A_2}$$

. (, B, C 가 $\overline{A_1A_2}$)

, $\triangle ABC$ $\overline{AB} + \overline{BC} + \overline{CA}$ 가 $가$ B, C $OX,$
 OY $\overline{A_1A_2}$. \diamond

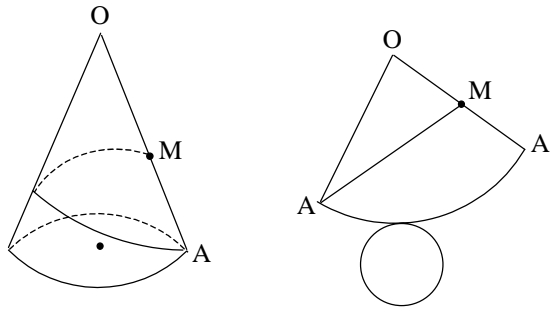
$\angle ABC$ P 가 . $\angle ABC = 90^\circ, \overline{BP} = 5$ P \overline{AB}
 \overline{BC} P .

3.3.2

3, 가 12 . OA
 OA M . A M OA
 ?



\overline{AM}



가 ,

$\angle AOA'$

$$12 \times \angle AOA' = 3 \times 360^\circ$$

90° . ,

$$AM = \sqrt{AO^2 + OM^2} = \sqrt{12^2 + 6^2} = 6\sqrt{5}$$

가 .



3, 가 9 .

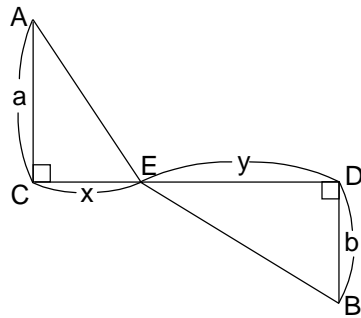
?

3.3.3

0 a, b , $x + y = a + b$ 가 $x > 0, y > 0$,

$$\sqrt{x^2 + a^2} + \sqrt{y^2 + b^2}$$

$\overline{AC} = a, \overline{EC} = x$ $\angle ACE$ 가 $\triangle ACE$ $\overline{BD} = b,$
 $\overline{ED} = y$ $\angle BDE$ 가 $\triangle BDE$.



$x + y = \overline{EC} + \overline{ED} = \overline{CD}$ 가 \overline{CD} E 가
 \overline{CD} $x + y = a + b$. ,

$$\sqrt{x^2 + a^2} + \sqrt{y^2 + b^2} = \overline{EA} + \overline{EB}$$

$$\overline{EA} + \overline{EB} \geq \overline{AB}$$
 ,

$$\sqrt{x^2 + a^2} + \sqrt{y^2 + b^2} \geq \overline{AB} = \sqrt{\overline{CD}^2 + (\overline{AC} + \overline{BD})^2} = \sqrt{(a + b)^2 + (a + b)^2}$$

. (, E 가 \overline{AB})
 $\sqrt{x^2 + a^2} + \sqrt{y^2 + b^2} \geq \sqrt{2}(a + b)$. \diamond

$x + y + z = a + b + c = 8$.

$$\sqrt{x^2 + a^2} + \sqrt{y^2 + b^2} + \sqrt{z^2 + c^2}$$

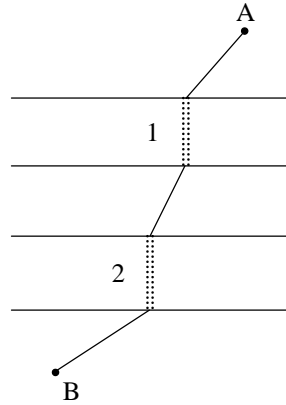


3.3.1

A B ,

. A B

가?



3.3.2

D

$ABCD$ 가

가

P

, $A, B, C,$

.



3.3.3

가

가

가?

가 1

가

가

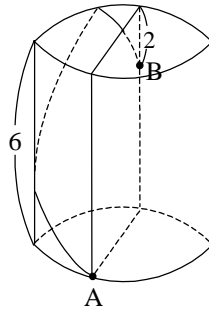
가

가 .



3.3.4

A B . , A
 B ,
 12 .



3.3.5

$$x_1 + x_2 + \dots + x_n = a, \quad y_1 + y_2 + \dots + y_n = b$$

$$\sqrt{x_1^2 + y_1^2} + \sqrt{x_2^2 + y_2^2} + \dots + \sqrt{x_n^2 + y_n^2}$$



3.3.6

x, y ,

$$|x - y| \geq |x| - |y|$$

[Redacted]

3.3.1

5

[Redacted]

3.3.2

가 .

3.4 가

x , x 가 x ,
 $[x]$ $[x]$. $f(x) = [x]$
 , 가 (Gauss function) (floor function)
 . x $x - [x]$ x , $\{x\}$
 .

가 가 $[x]$.

?

1

- (1) $[7] = ?$, $\{7\} = ?$
- (2) $[2.6] = ?$, $\{2.6\} = ?$
- (3) $[-4.8] = ?$, $\{-4.8\} = ?$

1

- (1) $[7] = 7$, $\{7\} = 0$
- (2) $[2.6] = 2$, $\{2.6\} = 0.6$
- (3) $[-4.8] = -5$, $\{-4.8\} = 0.2$

◇

3 $[x]$ 가 x 가
 . x 가
 $[x]$ (ceiling function)

$$[7] = 7, \quad [2.6] = 3, \quad [-4.8] = -4$$

가

가

- (1) $[x]$
- (2) $x = [x] + \{x\}$
- (3) $[x] \leq x < [x] + 1, \quad x - 1 < [x] \leq x$
- (4) $0 \leq \{x\} < 1$

. 가

[...]

'...'

2

x

$$2[x] - 3 > 0$$

2 $2[x] - 3 > 0$ $[x] > \frac{3}{2}$, $[x]$ $[x] =$
 $2, 3, 4, 5, \dots$ $\blacksquare x \geq 2$ \diamond

3

$$x - \left[\frac{2x}{3} \right] = 3$$

1 (2) $\left[\frac{2x}{3} \right] \leq \frac{2x}{3} < \left[\frac{2x}{3} \right] + 1$

$$2 = x - \left[\frac{2x}{3} \right] - 1 < x - \frac{2x}{3} \leq x - \left[\frac{2x}{3} \right] = 3$$

$$2 < \frac{x}{3} \leq 3, \quad 6 < x \leq 9 \quad \cdot \quad \left[\frac{2x}{3} \right] = 4 \quad 5 \quad 6$$

(1) $\left[\frac{2x}{3} \right] = 4 \quad : 4 < x < 7.5 \quad \cdot \quad x - 4 = 3, \quad x = 7.$

(2) $\left[\frac{2x}{3} \right] = 5 \quad : 7.5 \leq x < 9 \quad \cdot \quad x - 5 = 3, \quad x = 8.$

(3) $\left[\frac{2x}{3} \right] = 6 \quad : \quad x = 9 \quad \cdot$

... $\blacksquare x = 7, 8, 9.$ \diamond

2 $\left[\frac{2x}{3} \right] = 3 \quad \cdot \quad x = \left[\frac{2x}{3} \right] + 3 \quad x \quad \cdot$

1 $6 < x \leq 9 \quad x = 7, 8, 9$ 가 \cdot
 \diamond

가

가 1

(1) $y = [x] \quad 1$

(2) $y = [x] \quad \cdot \quad x \leq y \quad [x] \leq [y] \quad \cdot$

(3) $x, y \quad , [x] + [y] \leq [x + y] \quad \cdot$

가

2

- (1) $x \in \mathbb{R}, a \in \mathbb{Z} \quad [x + a] = [x] + a$
- (2) $a, b \in \mathbb{Z}, a > 0, b = qa + r, 0 \leq r < a \quad \left[\frac{b}{a}\right] = q$.
- (3) $a, b \in \mathbb{Z}, a > 0, b > 0, 1, 2, \dots, b \quad a \quad \left[\frac{b}{a}\right]$.
- (4) $x \in \mathbb{R}, a \in \mathbb{Z}, a > 0, \left[\frac{[x]}{a}\right] = \left[\frac{x}{a}\right]$. , $a, b, c \in \mathbb{Z}, a, b > 0, \left[\frac{\left[\frac{c}{b}\right]}{a}\right] = \left[\frac{c}{ab}\right]$.



(1)

(2) $\frac{b}{a} = q + \frac{r}{a}, 0 \leq \frac{r}{a} < 1 \quad \left[\frac{b}{a}\right] = q$.

(3) $1, 2, \dots, b \quad a \quad q, qa \leq b < (q + 1)a, q \leq \frac{b}{a} < q + 1$,

$\left[\frac{b}{a}\right] = q$.

(4) $x = [x] + \{x\}, [x] = qa + r, 0 \leq r \leq a - 1$,

$x = qa + (\{x\} + r), 0 \leq \{x\} + r < a$

, (2) , $\left[\frac{x}{a}\right] = q = \left[\frac{[x]}{a}\right]$.

□

가

, 가

?

4

- (1) 1 500 7 ?
- (2) 1 199 7 ?

$$\bullet \quad (1) \left[\frac{500}{7} \right] = \left[71 + \frac{3}{7} \right] = 71 \quad (2) \left[\frac{199}{7} \right] = \left[28 + \frac{3}{7} \right] = 28 \quad \diamond$$

5

	가		?
(1) 176	545	13	가?
(2) 248	794	7	가?
(3) 432	632	11	가?

3.4.1

x .

$$4[x]^2 - 36[x] + 45 < 0$$

● $4[x]^2 - 36[x] + 45 < 0$

$$(2[x] - 3)(2[x] - 15) < 0$$

· ,

$$\frac{3}{2} < [x] < \frac{15}{2}$$

, $[x]$

$$[x] = 2, 3, 4, 5, 6, 7. \dots \blacksquare 2 \leq x < 8$$

◇

○ $[a]$ a .

$$[x]^2 - 3[x] + 2 = 0, \quad [y]^2 - 7[y] + 12 = 0$$

x, y $x + y$ 가 .

3.4.2

$$x^2 - [x^2] = x - [x] \quad (0 < x < 2)$$

$$x^2 - [x^2] = x - [x] \quad (0 < x < 2)$$



$$0 < x < 2 \quad []$$

$$[x] = 0 \quad 0 < x < 1 \quad [x] = 1 \quad 1 \leq x < 2$$

$$[x^2] = 0 \quad ? \quad 0 < x < 2$$

$$0 < x^2 < 4$$

$$0 < x^2 < 1, \quad 1 \leq x^2 < 2, \quad 2 \leq x^2 < 3, \quad 3 \leq x^2 < 4$$

.



$$(1) \quad 0 < x < 1 \quad : [x] = 0, [x^2] = 0. \quad x^2 - 0 = x - 0. \quad x = 1.$$

$$(2) \quad 1 \leq x < \sqrt{2} \quad : [x] = 1, [x^2] = 1 \quad x^2 - 1 = x - 1. \quad x = 0 \text{ or } 1. \\ x = 1$$

$$(3) \quad \sqrt{2} \leq x < \sqrt{3} \quad : [x] = 1, [x^2] = 2 \quad x^2 - 2 = x - 1. \quad x = \frac{1 \pm \sqrt{5}}{2}. \\ x = \frac{1 + \sqrt{5}}{2}$$

$$(4) \quad \sqrt{3} \leq x < 2 \quad : [x] = 1, [x^2] = 3 \quad x^2 - 3 = x - 1. \quad x = 2 \text{ or } -1.$$

.

$$(1)-(4) \quad x = 1, \frac{1 + \sqrt{5}}{2} \quad \dots \quad \blacksquare \quad \diamond$$



$$2[x^2] - [x] = 2 \quad (1 < x \leq 2)$$

3.4.3

 x 가 $[100x]$

$$\left[x + \frac{1}{100} \right] + \left[x + \frac{2}{100} \right] + \cdots + \left[x + \frac{19}{100} \right] = 2000$$

19

$$\frac{1}{100}, \frac{2}{100}, \cdots, \frac{19}{100}$$

1

 $[x]$ $[x+1]$

$$\left[x + \frac{a}{100} \right] = k, \quad (1 \leq a \leq m)$$

$$\left[x + \frac{b}{100} \right] = k+1, \quad (m+1 \leq b \leq 19)$$

$$mk + (19 - m)(k + 1) = 2000$$

$$, 19k = m + 1981 \quad . \quad 0 \leq m \leq 19 \quad , \quad 1981 \leq 19k \leq 2000 \quad .$$

$$104.2 < k < 105.3 \quad k = 105 \quad m = 14 \quad .$$

$$\left[x + \frac{14}{100} \right] = 105, \quad \left[x + \frac{15}{100} \right] = 106$$

$$, x \quad 105.85 \leq x < 105.86 \quad . \quad \cdots \quad \blacksquare \quad 10585 \quad \diamond$$

 x 가 $[100x]$

$$\left[x + \frac{19}{100} \right] + \left[x + \frac{20}{100} \right] + \cdots + \left[x + \frac{54}{100} \right] = 321$$



3.4.1

$$x \quad .$$

$$7[x] - 2 > 0$$



3.4.2

$$x \quad .$$

$$3[x]^2 - 7[x] - 6 > 0$$



3.4.3

$$[x] = 1, [y] = 2, [z] = -3 \quad , [x + y - z]$$

$$. \quad , [x] \quad x \quad .$$



3.4.4

$$.$$

$$2x - [x] = 2$$



3.4.5

$$.$$

$$2x^2 - [x] = 2$$



3.4.6

$$[x] + [1 - x] \quad .$$



3.4.7

$[x] - [x] = 0$ 가?



3.4.8

$[a] = a$. x ,
 y 가 , x 가 가 ,
 $x + y$ 가 .

$$y = 2[x] + 3, \quad y = 3[x - 2] + 2$$



3.4.9

x 가 0 0 , x 가 0 1
 . , 가 .



3.4.10

$$x - [x^2] = x^2 - 5[x] \quad (2 < x < 4)$$



3.4.11

$$x$$

$$2[x]^2 - 7[x] - 8 > 0$$



3.4.12

$$[x + y] = [x] + [y] + [\{x\} + \{y\}] \quad . \quad , [x] \quad x$$

$$, \{x\} \quad x - [x] \quad .$$



3.4.13

$$a \quad b \quad a, b$$

가 .



3.4.14

$$a_n \quad \text{가} \quad [] \quad .$$

$$a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 2, a_5 = 3, a_6 = 3, a_7 = 4, \dots$$



3.4.15

$$[\sqrt{[x]}] = [\sqrt{x}] \quad \text{가} \quad .$$



3.4.16

$$[a + b + c] \geq [a - 2d] + [b + d] + [c + d] \quad \text{가}$$

.

3.4.1

$$\left[\frac{[m\alpha]n}{\alpha} \right]$$

, $m \ n$, $\alpha \ n$

3.4.2

$$[x] + [y] + [x + y] \leq [2x] + [2y]$$

3.4.3

 m, n

$$n = \left[\frac{n}{m} \right] + \left[\frac{n+1}{m} \right] + \cdots + \left[\frac{n+m-1}{m} \right]$$

3.4.4

2 3 5 6 7 8 10 11 12 13 14 15 17 18 ...

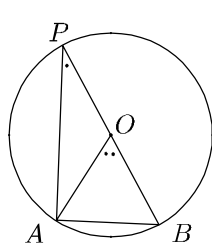
$$\left[n + \sqrt{n} + \frac{1}{2} \right]$$

4

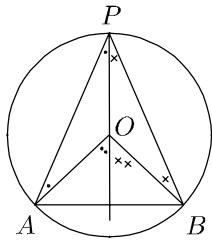
4.1

가

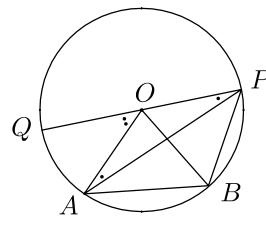
가



(1)



(2)



(3)

■ $\angle APB$ 가 (1), $\angle APB$ 가

(2) $\angle APB$

가 (3)

$\triangle OAP$ $\angle QOA = 2\angle OPA$, $\triangle POB$ $\angle QOB = 2\angle OPB$

$$\angle AOB = \angle QOB - \angle QOA = 2\angle OPB - 2\angle OPA = 2\angle APB$$

가 AB AB $\frac{1}{2}$



90°

1

가

가

가

가

,

$1/2$

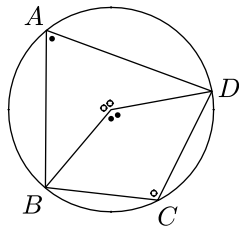
가

□

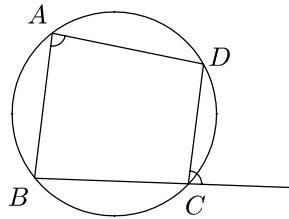
(1)

180°

(2)



(4)



(5)

(4) (5)

가

(1)

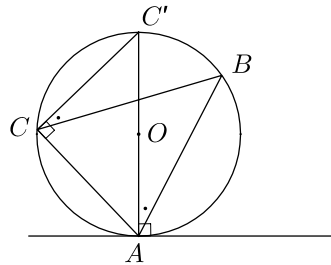
(1) AB P, Q , $\angle APB = \angle AQB$
 , A, B, P, Q .

(2) 180° .

(3) 가 .

Blank space for student work.

● A C' , $C C'$ $\angle C'CA = \angle R$



$\angle BCA = 90^\circ - \angle C'CB$. $\angle C'AT = 90^\circ$ $\angle BAT =$
 $90^\circ - \angle C'AB$. $\angle C'CB =$
 $\angle C'AB$. $\angle BAT = \angle BCA$ 가 . \square

? (5) B 가 C 가 C
 . A CD BC C
 ?

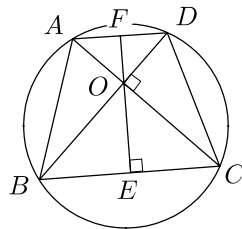
가

(Brahmagupta 598–660?)

2

()

$ABCD$, O
 BC OE BC AD



AB $\angle ADB = \angle ACB$ $\triangle OEC$
 $\angle ECO + \angle EOC = \angle R$, $\triangle OBC$ $\angle BOE + \angle EOC = \angle R$
 $\angle ECO = \angle BOE$ $\angle BOE = \angle FOD$
 $\angle FOD = \angle FDO$ $FD = FO$
 $AF = FO$
 $AF = FD$ \square

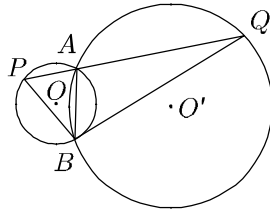
$P'Q'$
 PBQ $P'BQ'$
 $\angle PBQ = \angle P'BQ'$

(Möbius 1790–1868)가 1827

3

()

O, O' A, B A O, O'
 P, Q $\angle PBQ$



\square AB AB $\angle APB$ $\angle AQB$
 $\therefore \triangle PBQ$

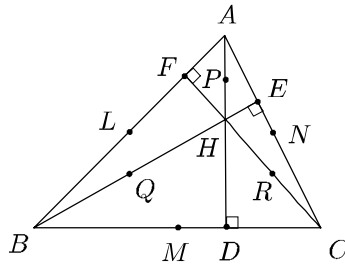
$$\angle APB + \angle AQB + \angle PBQ = 180^\circ$$

$$\angle PBQ = 180^\circ - (\angle APB + \angle AQB) \quad \therefore \angle APB \quad \angle AQB \text{가}$$

$\angle PBQ$ \square

4

((Euler)가 9)
 $\triangle ABC$ D, E, F L, M, N H
 P, Q, R , 9 .



\square $\triangle ABC$ $LN \parallel BC$ $LM = \frac{1}{2}AC$, $\triangle ADC$

$$NA = NC = ND = \frac{1}{2}AC$$

\therefore , $\square LMDN$.
 180° D, L, M, N . $\triangle AHC$ $PN \parallel HC$
 $\triangle ABC$ $AB \parallel MN$

$$\angle AFC = \angle PNM = \angle R$$

$$\angle PNM = \angle PDM \quad D, M, N, P$$

□



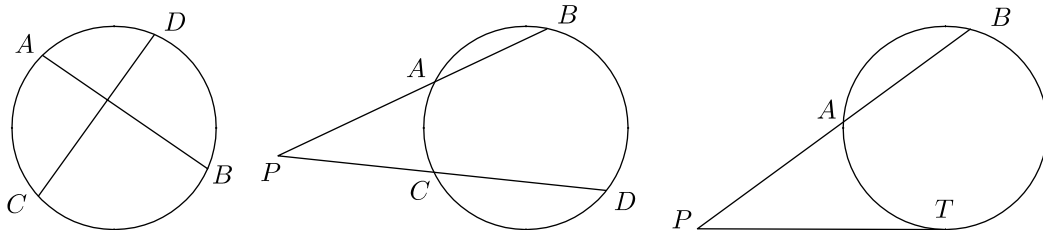
(1)

(2) AB, CD P $PA \cdot$

$$PB = PC \cdot PD \text{ 가}$$

(3) P

$$T, A, B \quad PT^2 = PA \cdot PB \text{ 가}$$



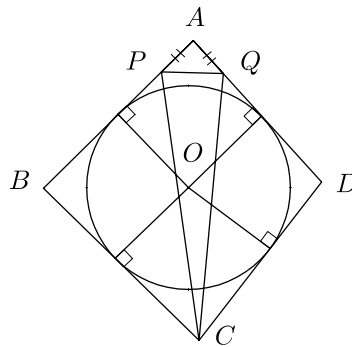
(2)

AB, CD 가 () P $PA \cdot PB = PC \cdot PD$
 , A, B, C, D .

5

()
 $ABCD$ $AB + CD = BC + DA$,

BA BA $BC = BP$ P
 AD $DC = DQ$ Q $CP,$
 CQ, PQ



가 $AB - BC = DA - CD$ $AP = AQ$. $\triangle APQ, \triangle BCP,$
 $\triangle DCQ$. $\angle A, \angle B, \angle D$ $PQ,$
 CP, CQ , $\triangle PCQ$ O . O $AB,$
 BC, CD, DA E, F, G, H .
 $OE = OF = OG = OH$. ()
 .) O , OE $\square ABCD$

□

4.1.1

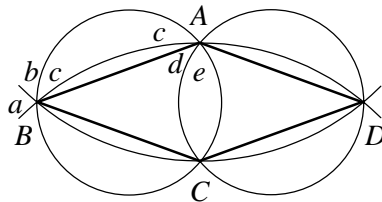
(1903 가 Eötvös) A, B, C, D ,
 k_1 B, C, D , k_2 A, C, D , k_3
 A, B, D , k_4 A, B, C . B
 k_1 k_3 C k_2 k_4
 .



()



a, b, c, d, e . (k_3 k_4
 c .)



b c
 , d
 c . $b = c = d$, $a + b + c =$
 $c + d + e = 180^\circ$ $a = e$. , k_1 k_3
 k_2 k_4 가 가 . \square

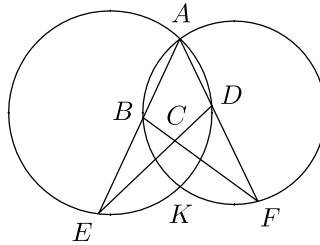


(1904 가 Eötvös)

4.1.2

(, 4)

EA, ED FA, FB 가 $\triangle AED, \triangle ABF,$
 $\triangle BEC, \triangle CFD$ $\triangle AED, \triangle ABF$ K



$\angle EAK = \angle EDK$ EK , $\angle EAK = \angle BFK$
 BK . , $\angle CDK = \angle CFK$ C, D, F, K
 $\triangle CDF$ K . $\triangle BCE$
 K . □

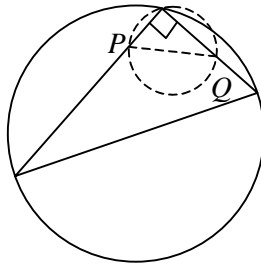


()

4.1.3

(1894 가 Eötvös) P, Q 가 .
 P ,
 Q . P, Q 가 가
 가?

O ABC , AB, AC P
 Q 가 . BC 가 A 가 , PAQ
 , A PQ M
 , P, Q O .



, O P, Q , PQ M O
 A . AP, AQ 가 O B, C
 ABC 가 .
 가 O P, Q
 M O . ,
 , P, Q 가 O $R \leq r + d$. , R
 O , r M MP , d
 MO . \diamond

(1895 가 Eötvös) ABC 가 ,
 N NBC, NCA, NAB 가



4.1.1

‘ (1)’ (2) . . ,
180°



4.1.2

‘ (2)’ . . , AB,
CD P PA · PB = PC · PD
, A, B, C, D .



4.1.3

P, Q가 X, Y . X
P, Q A, D , Y P,
Q B, C . ABCD



4.1.4

P, Q가 X . X
P, Q A, C; B, D . AB//CD



4.1.5

ABC AC O BC
D . , AC DE O
E . ADE .



4.1.6

(1971) DEB ,
 $DE = 3$ $EB = 5$. O .
 OE C . $EC = 1$



4.1.7

(1) $ABCD$ $AB + CD = BC + DA$.
 (2) $ABCDEF$ 가 $AB + CD + EF = BC + DE + FA$ 가



4.1.8

(1999) P, Q, R, S 가
 PQ SR $QR = SR$
 T , QT 가
 RQT 가
 (1) $PS = QR$,
 (2) PQT QR, QS 3 .



4.1.9

180°

4.1.10

(1998 Intermediate) 10,
 17 X, Y $XY = 16$.
 XY . PXQ 가
 P, Q . PQ 가
 가

4.1.11

AB C AB, AC, CB
 k_1, k_2, k_3 . k_2
 k_3 k_2, k_3
 D, E . C AB
 k_1 F . $FDCE$ 가

4.1.12

ABC AC AC
 P , BP 가
 $\angle ABC$.

4.1.13

(1998 Senior Contest) $ABCD$
 AC BD 가 M . AB N ,
 NP CD 가 CD P .
 M, N, P .

4.1.1

$ABCD$, k_1 B, C, D , k_2
 A, C, D , k_3 A, B, D , k_4
 A, B, C B k_1 k_3
 C k_2 k_4

4.1.2

(1973, 1976, 1988) O O
 AB 가 XY 가
 AX BY P

4.1.3

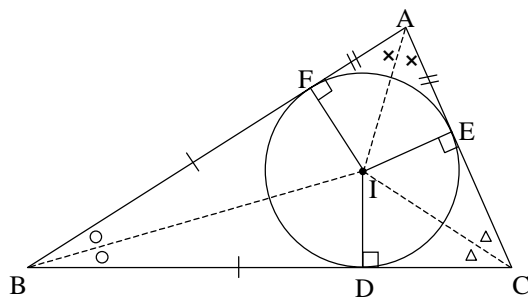
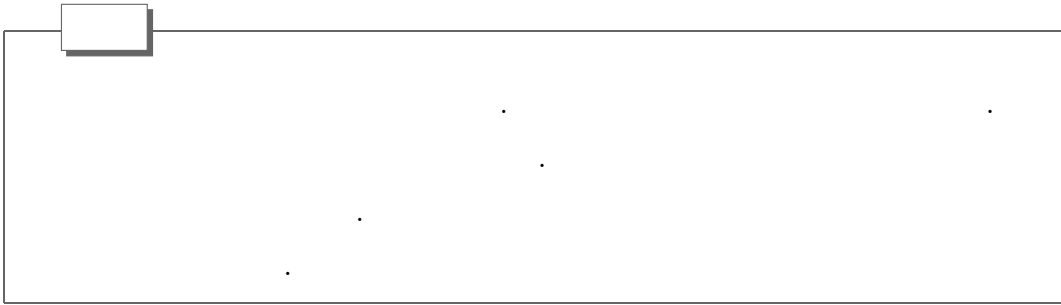
P, Q P AB
 $AP \cdot PB$ 가 AB

4.1.4

(1917 가 Eötvös) A B k
 A B k
 ()

4.2

가



ABC I D, E, F 가
 I ID, IE, IF I 가

(1)

I . $\triangle AFI \cong \triangle AEI$
 $FI = EI, \angle AFI = \angle AEI = 90^\circ, AI$
 . , $\angle FAI = \angle EAI$, $AI \perp \angle A$
 . 가 $BI, CI \perp \angle B, \angle C$.

(2)

가 가 가 .

‘ 가?’

‘ 가?’

?

1





AI 가 $\angle A$ 의 이등분선이고, I 는 $\angle B, \angle C$ 의 이등분선의 교점이다. I 에서 BC, CA, AB 에 수직 선분을 D, E, F 로 놓는다.

$$\triangle BDI \equiv \triangle BFI, \quad \triangle CDI \equiv \triangle CEI$$

$$, FI = DI = EI \text{ 가}$$

$$\triangle AFI \equiv \triangle AEI$$

, AI 가 $\angle A$ 의 이등분선이다. □

$\triangle ABC$ 의 넓이를 S 라 하고, a, b, c 는 $\angle A, \angle B, \angle C$ 의 대변의 길이, r 은 내접원의 반지름이다.

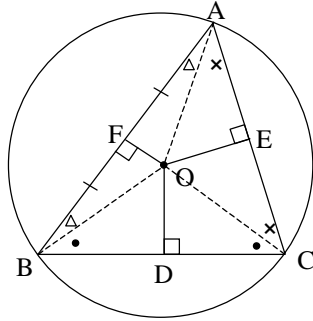
$$S = \frac{1}{2}r(a + b + c) = rs$$

이므로, $s = \frac{a + b + c}{2}$ 이다.



I 에서 AB, BC, CA 에 수직 선분을 F, D, E 로 놓는다. $\triangle AFI, \triangle BDI, \triangle CEI$ 의 넓이를 각각 $\frac{1}{2}rf, \frac{1}{2}rd, \frac{1}{2}re$ 라 하면 $\triangle ABC$ 의 넓이는 $\frac{1}{2}r(a + b + c)$ 이다. □





ABC O . AO, BO, CO O
 가 . .

(1)

D, E, F . $\triangle AFO, \triangle BFO$. O
 $AO = BO, \angle AFO = \angle BFO = 90^\circ, FO$
 . $AF = BF$, O AB
 . 가 O BC, CA .

(2)

2



AB, AC

O

, O

BC

D

$BD = CD$

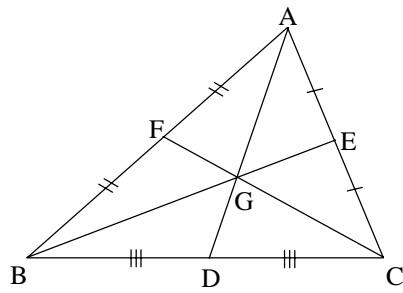
$$\triangle AFO \equiv \triangle BFO, \quad \triangle AEO \equiv \triangle CEO$$

$BO = AO = CO$ 가 ,

$$\triangle BDO \equiv \triangle CDO$$

, $BD = CD$.

□



가

3

\square . BC, AC D, E AD, BE
 G . $CG \perp AB$. AD 가

$$|\triangle ABD| = |\triangle ADC|, \quad |\triangle GBD| = |\triangle GDC|$$

$$, |\triangle ABG| = |\triangle ACG| \quad . \quad \text{가} \quad |\triangle ABG| = |\triangle BCG| \quad ,$$

$$|\triangle ACG| = |\triangle BCG|$$

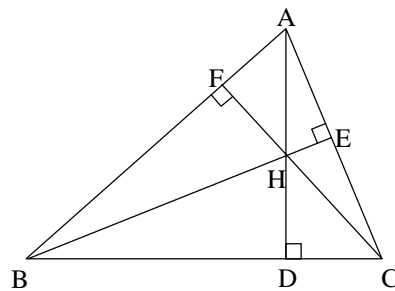
. , CG $AF = BF$. \square

4

2 : 1 .

\square $\triangle ABG$ $\triangle BDG$ 가 2 : 1 AG :
 $GD = 2 : 1$. \square

■

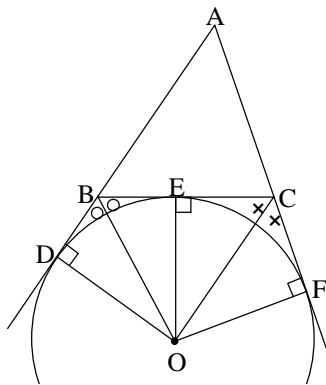


5

가 . $AD \perp BE$
 H AB F , F, H, C 가
 , CH
 AB F $CF \perp AB$.
 . A, B D ,
 E H . $CH \perp AB$
 . $\angle AEB = \angle ADB$ $\square ABDE$. 가
 $HDCE$,
 $\angle ABE = \angle ADE (= \angle HDE) = \angle HCE (= \angle ACF)$
 $\triangle ABE \cong \triangle ACF$ 가 . , $\angle AFC = \angle AEB = 90^\circ$.
 □



가 .



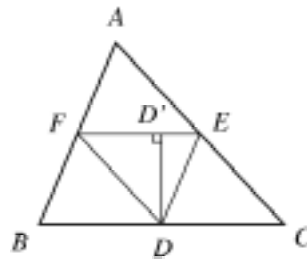
O AB, BC, CA D, E, F .

$$\triangle BOD \cong \triangle BOE, \quad \triangle COE \cong \triangle COF$$

$OD = OE = OF$ 가 . , O OD
 AB, AC BC . $\triangle ABC$
 $2AD$.

4.2.1

$\triangle ABC$ D, E, F $\triangle ABC$ $\triangle DEF$



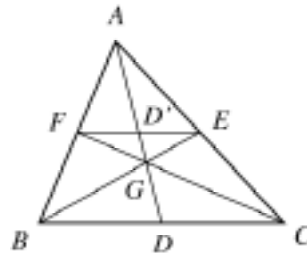
$DD' \perp EF$ $DD' \perp BC$ $BC \perp EF$,
 $DD' \parallel BC$.
 E, F DE E', F' 가
 EE', FF' AC, AB $\triangle ABC$ $\triangle DEF$

□

$\triangle ABC$ BC, CA, AB D, E, F .
 BC, CA, AB EF, FD, DE D', E', F'
 $\triangle ABC$ $\triangle D'E'F'$.

4.2.2

$\triangle ABC$ D, E, F . $\triangle ABC$ $\triangle DEF$



$AD \parallel EF \parallel D'E'$.

$$FD' = \frac{1}{2}BD, D'E = \frac{1}{2}DC, \quad , \quad FD' = D'E$$

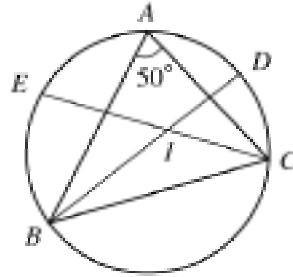
, $DD' \parallel EF \parallel D'E'$. 가

, $G \triangle DEF$. □

 $\triangle ABC$ A_1, B_1, C_1 $\triangle A_1B_1C_1$
 A_2, B_2, C_2 . $\triangle ABC, \triangle A_1B_1C_1, \triangle A_2B_2C_2$
 $\triangle ABC$.

4.2.3

$\angle EAD = ?$



, I , 1 .



$$\angle ABE = \angle ACE = \frac{1}{2}\angle C$$

$$\angle ABD = \frac{1}{2}\angle B$$

,

$$\angle EBD = \frac{1}{2}(\angle B + \angle C) = \frac{1}{2}(180^\circ - \angle A) = \frac{130^\circ}{2}$$

. $\angle EAD = 2\angle EBD = 130^\circ$. , $\angle EAD$

$$2\pi \times \frac{130^\circ}{360^\circ} = \frac{13}{18}\pi$$

◇



가 1 $\triangle ABC$ 가

I . BI CI

, D, E ,

$\angle DIE = 110^\circ$ 가

. BE CD

?



4.2.1

$\angle A = 90^\circ$, $BC = a$, r , $\triangle ABC$



4.2.2

$\triangle ABC$ I , $\angle BIC$ $\angle A$



4.2.3

$\triangle ABC$ I BC AB, AC
 D, E $\triangle ADE$ $AB, AC,$
 BC



4.2.4

ABC O , CO AB 가
 P, BO AC 가 Q $BP = PQ =$
 QC $\angle A = 60^\circ$



4.2.5



4.2.6

$\triangle ABC$

S

$$S = \frac{AB + BC - CA}{2} \times r$$

r $\angle B$

$\triangle ABC$



4.2.7

가?



4.2.8

$\triangle ABC$
 $\frac{AI}{ID}$

$I,$

AI

BC

D



4.2.9

n

n

가

, 가 T

$$\frac{1}{3}\sqrt{\sqrt{3}T}$$



4.2.10

ABC

$O,$

$H,$

AC

D

BO 가

ABC

E

H, D, E 가

4.2.1

 $\triangle ABC$ r_1, r_2, r_3 r

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$$

4.2.2

4.2.3

 $AB = AC$ ABC AB, AC P, Q $\triangle ABC$ PQ $\triangle ABC$

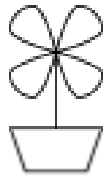
4.2.4

 $\triangle ABC$ $\triangle ABC$

5.1

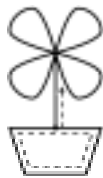
가

?



가

가



가 가

가 가

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가 가

3

가 가

1



가

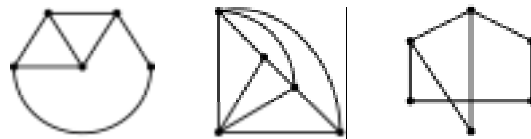
가

?



가 가

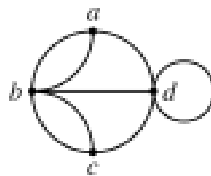
가



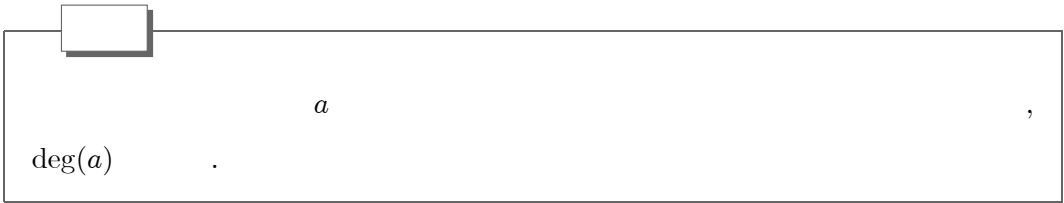
$$G = (V, E)$$

V , E

$$G = (V, E)$$



$$V = \{a, b, c, d\} \quad , \quad E = \{(a, b), (a, b), (b, c), (b, c), (a, d), (b, d), (c, d), (d, d)\} \\ (d, d)$$



$$\deg(a) = \deg(c) = 3, \deg(b) = \deg(d) = 5$$

가



1

2



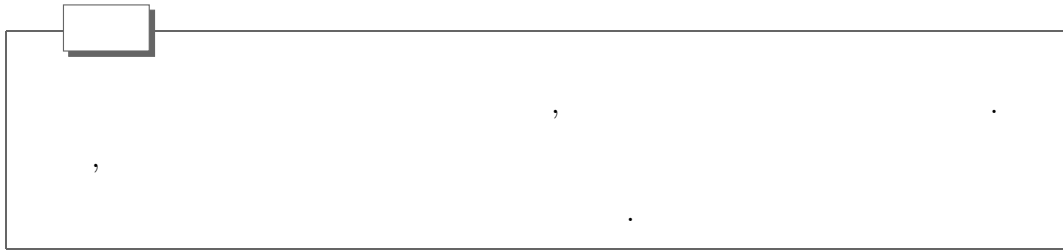
가

가



■ 가

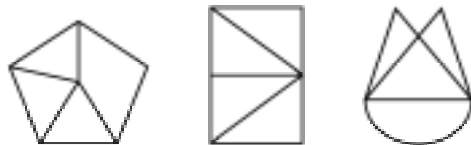
가 가 , 가
 가 가
 ? 가 가

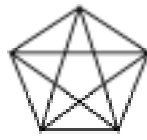


가 가
 가 . ()

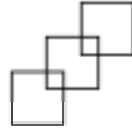


3

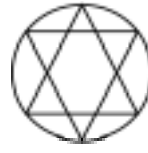




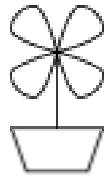
(a)



(b)



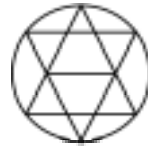
(c)



(d)



(e)



(f)

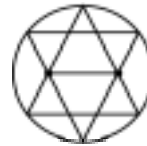
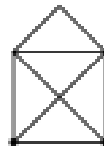
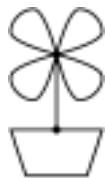
가 가

, (a), (b), (c)

가 가

(d), (e), (f)

가



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가 (,)

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가 가

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가 가

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가 .

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$$x_1 - x_2 - \cdots - x_n - x_1$$

.

가

, G

,

$$x_2, \dots, x_n$$

.

x_i

e

.

(=)

x_i

,

$$x_i - x_{i+1} - \cdots - x_n - x_1 - \cdots - x_i$$

x_i 가 e 가
 가 . 가 x_i
 가 가 . □

가



x, y , x, y
 가 가
 가 .
 가 . $x - y$ 가 가 ,

$$v_1 - \dots - v_m - x - y - w_1 - \dots - w_n - v_1$$

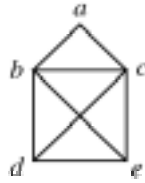
$$x - y$$

$$y - w_1 - \dots - w_n - v_1 - \dots - v_m - x$$

가 .
 가

. □

가 가 ?



$\deg(d) = 3, \deg(e) = 3$

가 .

$a - c - d - e - b - c - e$

$\deg(a) = 2, \deg(b) = 4, \deg(c) = 4,$

d, e . 가

, $d - b -$

. ◇

5.1.1

n n , 가 n

가 가



가 3 .

가 4가 .

가 가 가 ◇



가 가 .

5.1.2

0 6 가

28 가

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28 가

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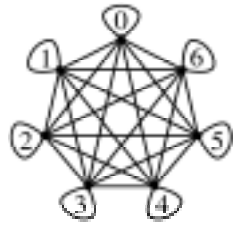
•••••••••••••

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••••••••••~

7 . 0 6 가

6 [1,6] , 1



가

가 가

6 가 가

◇

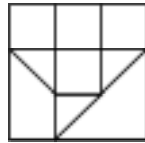
0 9 가 가?

가 가?

가

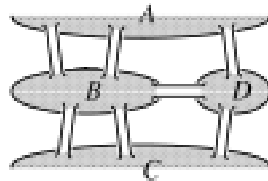
5.1.1

가 . (.) , . ?



5.1.2

가 . 가?



5.1.3

가 $x \neq y$, 가 $x \neq y \neq z \neq x$. 가 ? ‘ ’ , .



5.1.4

20

,

?

가?



5.1.5

가

가

.

a, b

.

$a \ b$

가

, a, b

가

.



5.1.6

n

가

,

가

,

.

.



5.1.7

n

n

가

.

.

, n

,

n

.

5.1.1

가 가 가 . 가
가 가 가 가
가?

5.1.2

가 $2k$ 가 .
' ' k
.

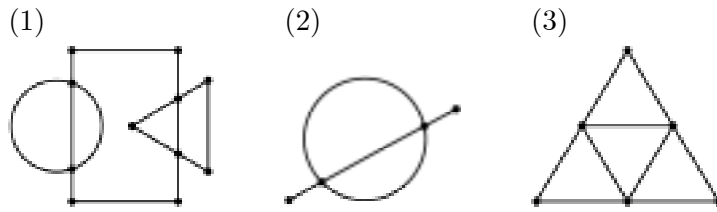
5.2

가 가
 , , () 가
 ?

1

$v, e,$

f



(1) $v = 11, e = 15, f = 5$

(2) $v = 4, e = 5, f = 2$

(3) $v = 6, e = 9, f = 4$

◇

v, e, f ?

2

$v - e + f$

()

$e,$ G f $v,$

$$v - e + f = 1$$



$$v - e + f = 1 - 0 + 0 = 1$$

(1)

(2)

가

$$v - e + f$$

(1)

, (2)

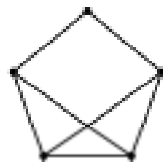
$$v - e + f$$

$$v - e + f = 1$$

□

가

?



3

가 20

30

가

가?

$v = 20, e = 30$. $v - e + f = 1$ $20 - 30 + f = 1$
 $f = 11.$ ◇

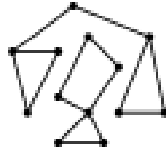
4

가 2 5 가 .

$e = 2v, e = f + 5$ $v - e + f = 1$.
 $\frac{1}{2}e - e + (e - 5) = 1$
 $e = 12, v = 6.$ ◇

5

$v - e + f$.



?

가 ?



()

G 가 k 가 , , v, e, f

$$v - e + f = k$$

$v_i - e_i + f_i = 1$ $(i = 1, \dots, k)$

$$(v_1 + \dots + v_k) - (e_1 + \dots + e_k) + (f_1 + \dots + f_k) = k$$

v 가 $(v_1 + \dots + v_k)$, e 가 $(e_1 + \dots + e_k)$, f 가 $(f_1 + \dots + f_k)$ 이므로

$$v - e + f = k$$

□

6

3 가 4, 10 가?

5.2.1

(: 가) 가
 가 . , , 가 v, e, f

$$v - e + f = 2$$

.



.

가

$$v - e + f = 1$$

,

f 가 1

,

$$v - e + f = 2$$

.

□



가 k

.

$$v - e + f = k + 1$$

.

5.2.2

가 8 가 . 3
가?
.



2 가 . .



가 3 . . ,

$$2e = 3v = 24$$

$$e = 12$$

$$v - e + f = 1$$

$$, 8 - 12 + f = 1$$

$$f = 5$$



12

20

. , , .



5.2.1

12

3

20

10

12

, ,

, $v - e + f = 2$ 가

5.2.2

 $v = e + 1$ 

5.2.3

가 n

가

,

 k n k 

5.2.4

가

 A, B 가 $v - e + f$ a, b n C c , a, b, c



5.2.5

$f = 0$

가

$v - e +$



5.2.6

k

, ,

(,

) v^+, e^+, f^+

$v^+ - e^+ + f^+ = 2$



5.2.7

가

8

,

7

?

5.2.1

 n
 $e + f$

.

가

 $v -$

5.2.2

3

.

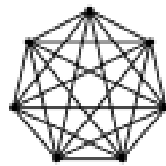
 $v - e + f$ 가

.

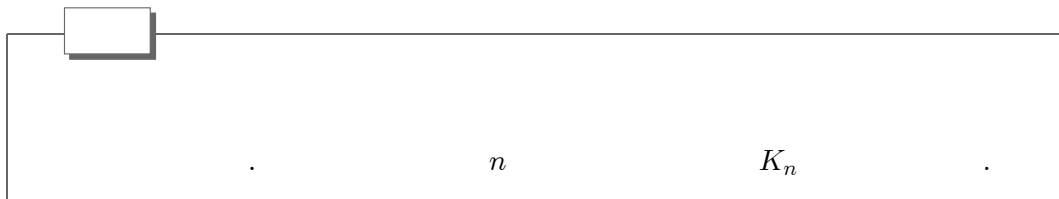
가

5.3 가

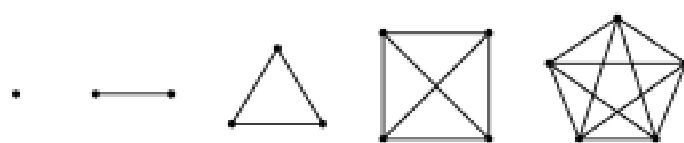
가
가 가



가 ?



$n = 1, 2, 3, 4, 5$



1

$n = 6$

K_6

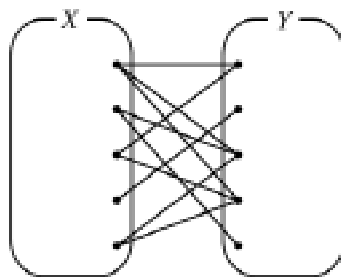


가

5

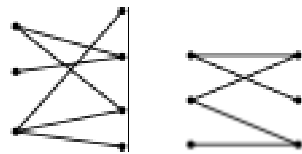
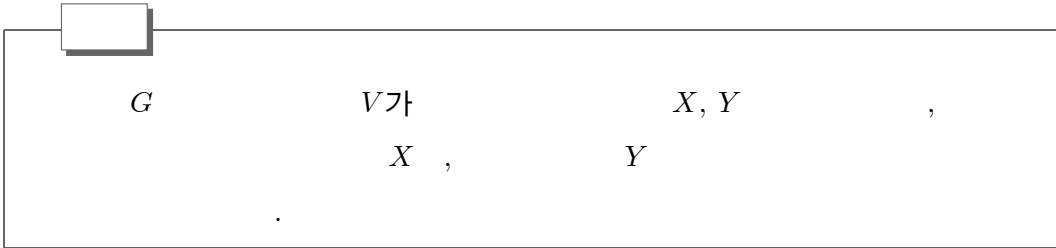
: , ?
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 : , 5 X , Y .

가 ?



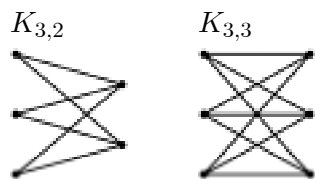
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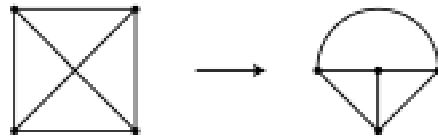


3 4

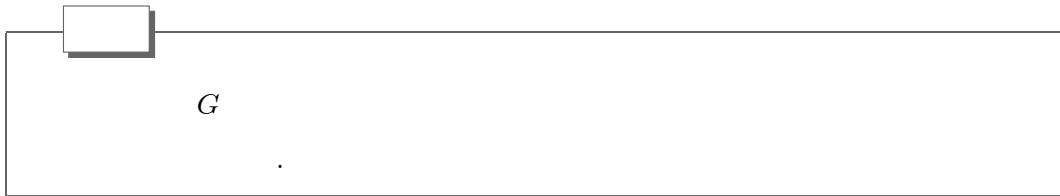
가 m , $K_{n,m}$ 가 n ,



, 가 ,
 가 가 K_4
 가 가

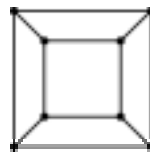
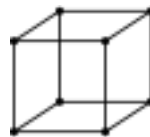


K_4



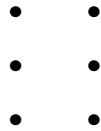
2

가?

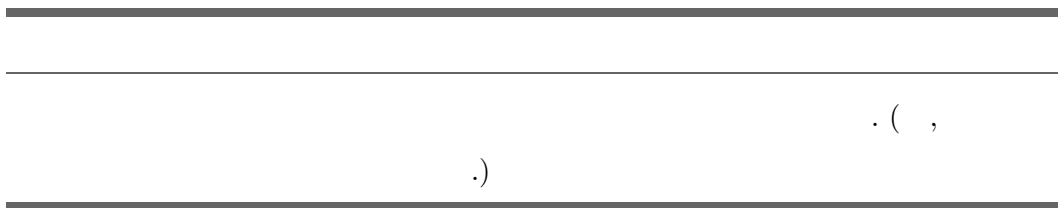
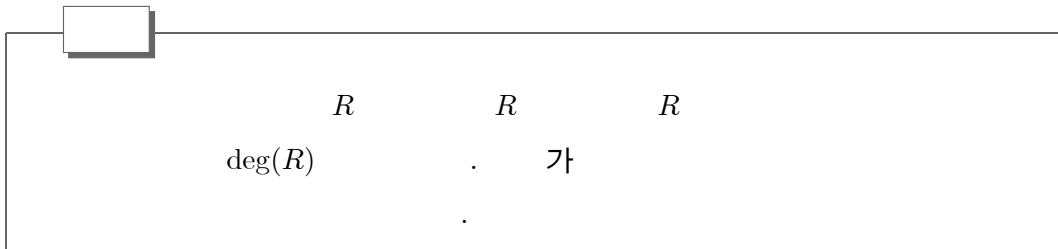


3

, , . A, B, C ,
 ,
 가?



? ' $K_{3,3}$ ' 가
 . $K_5, K_{3,3}$.



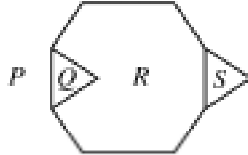
가 1
 2가

2가
 2 .

□

4

$$\deg(P) = 9, \deg(Q) = 3, \deg(R) = 9, \deg(S) = 3 \quad .$$



$$3 + 9 + 3 + 9 = 24$$

12 가 .

G 가 가 v ,
 e $e \leq 3v - 6$.

■

f R_1, R_2, \dots, R_f .
 가 .
 3 .

$$2e = \deg(R_1) + \deg(R_2) + \dots + \deg(R_f) \geq 3f$$

$$f = 2 - v + e$$

가 . □

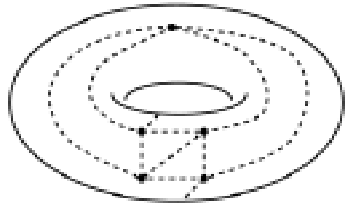
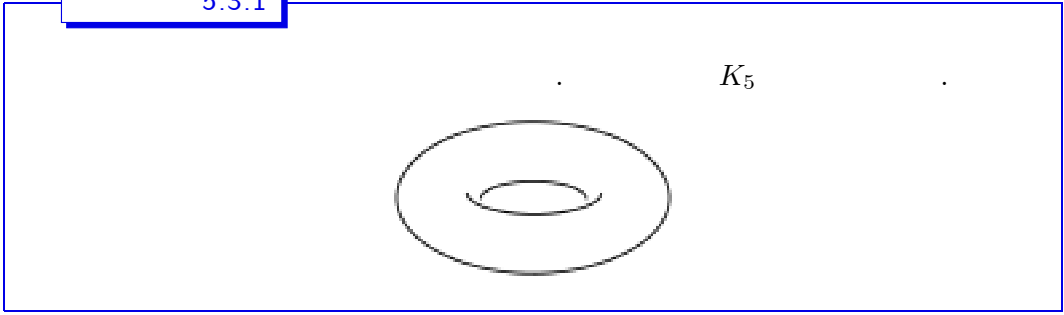
K_5 .

5

K_5 가 가 .

$3v - 6 = 9$. K_5 5 10 가 . $e = 10$
 $e \leq 3v - 6$ K_5 가
□

5.3.1



$K_{3,3}$

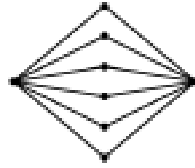
5.3.2

 i $K_{2,i}$. $K_{2,i}$

2

 i

.



◇



가

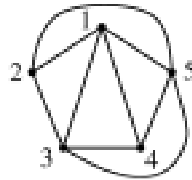
.

5.3.3

G_n 가 .
 (1) $\{v_1, v_2, \dots, v_n\}$ 가 .
 (2) $\gcd(i, j) = 1$ v_i, v_j .
 G_n 가 n .



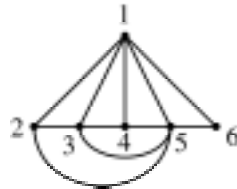
G_5



가 G_6, G_7 $n \geq 6$ 가 .



$n = 6$



$n \geq 7$ 가 v_1, v_2, v_3, v_5, v_7 .
 i, j K_5 ,
 가 가 가 .
 G_n 가 $n \geq 6$. \diamond



$G_{3,n}$

(1) $\{u_1, u_2, u_3; v_1, v_2, \dots, v_n\}$ 가 .
 (2) $\gcd(i, j) = 1$ u_i, v_j .
 $G_{3,n}$ 가 n .



5.3.1

k , A 가 B B A
 $, k \geq 1$.



5.3.2

v 가 $e = 3v - 6$.



5.3.3

4 가
 , .



5.3.4

가 가



5.3.5

10 가 가
 가?



5.3.6

가 . , ,
 v, e .

$$e \leq \frac{5}{3}v - \frac{10}{3}$$



5.3.7

가 7 가 가?



5.3.8

$v(\geq 3)$ e 가
 $e \leq 2v - 4$.



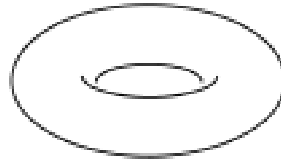
5.3.9

. $K_{3,3}$ 가 .

5.3.1

가?

K_7



5.3.2

K_5

가 가?

5.3.3

T 가

,

1

n

n

가

T 가
가

?

